PHY 132 – Summer 2000
LAB 4: Electric Power – DC Circuits[1]

Introduction

In the first part of this lab we look at electric power dissipated in a load resistor in a circuit with a real power source (finite internal resistance). We'll see that the power you can extract from a real source is maximized for a particular value of load resistor. A real-life example is matching loudspeakers to an audio amplifier (8 ohms standard). A mechanical analog is matching a propeller (load resistor) to a motor-boat (power source)– boat speed is optimized for proper pitch. In the second part of this lab we gain experience with the DC circuit rules (called Kirchhoff's Laws) by making null measurements using the famous Wheatstone Bridge circuit. Null measurements take many related forms, far beyond this particular circuit. We will see that they provide a way to make measurements that are much more precise than the instruments. How can this be? - see below.

1. DC Power

Theory

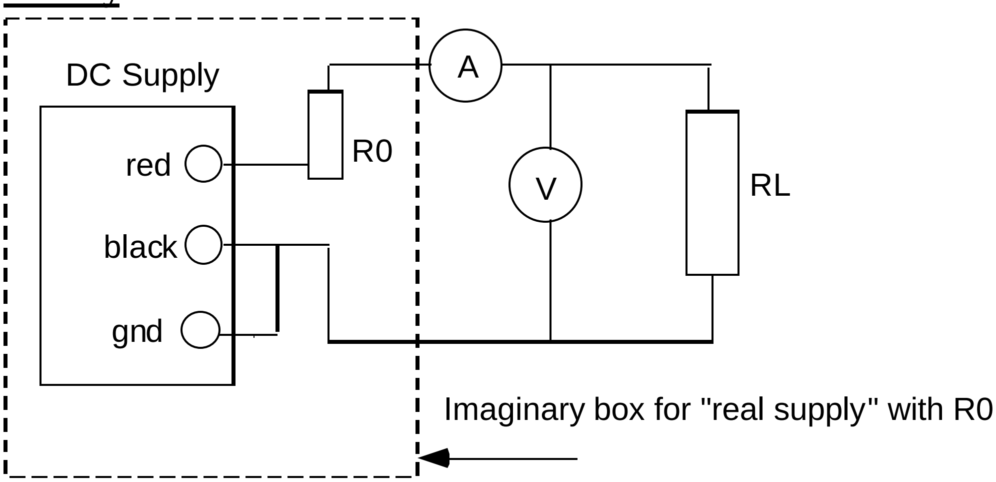

Figure 1. Circuit for measuring power in load resistor.

In Figure 1 we show a real power source with voltage V_0 and internal resistance R_0. The actual power supply has a very low internal resistance, so we

[1] Adapted by R. J. Jacob from P. Bennett, PHY-132 Lab Manual© (ASU)

add R_0 externally, but consider the combination as a "black box". This combined supply + R_0 is connected to a load resistor R_L. The meters are also shown in this case. Note that the current meter is in series with the load (the same current flows through both), while the voltmeter is in parallel (the same voltage is across both). The power dissipated in the load resistor is given by

$$P(R_L) = IV = \left(\frac{V_0}{R_0 + R_L}\right)\left(\frac{V_0 R_L}{R_0 + R_L}\right) = \left[\frac{V_0}{R_0 + R_L}\right]^2 R_L \qquad \text{Eq. 1}$$

Note the structure of this equation (see your data): for small R_L we get a lot of current but small voltage, and vice-versa. Hence one "suspects" that there will be a maximum power at some intermediate value of R_L.

Procedure

1. Measure R_0 directly using the DMM on ohm range. The resistor must be removed from the circuit!
2. Connect the circuit in Fig. 1. See the earlier handout on "meters and wiring tips".
3. Measure (I,V) as you step through values of R_L. Use log-spaced (ie factors of 2) values in the range 10-1000 ohms. Take one set of data for increasing R_L then repeat values for decreasing R_L. This will serve to establish error bars.

Analysis

1. Show (calculus) that $P(R_L)$ has a maximum at $R_L = R_0$.
2. Plot $P(R_L)$ in GA. This is a non-linear plot, hence requires writing your own function. Determine V_0 and R_0 and errors from an optimized non-linear fit. . See "Fits" handout for procedure.

2. The Wheatstone Bridge

Theory

The Wheatstone bridge circuit is shown in figure 1. This is normally configured with adjustable resistors in the “arms”, one resistor as an unknown (say R4) to be determined and a meter to sense the voltage V5 across R5. It is powered by a constant voltage V_0. The bridge is balanced by adjusting resistor values until V5=0. This is determined by the following equation

$$V5=0 \text{ or } V_{bd} = V_{cd} \qquad \text{Eq. 1}$$

This can be written in terms of resistor values using the voltage divider relation,

$$V_0R_2/(R_1+R_2) = V_0R_4/(R_3+R_4) \qquad \text{Eq. 2}$$

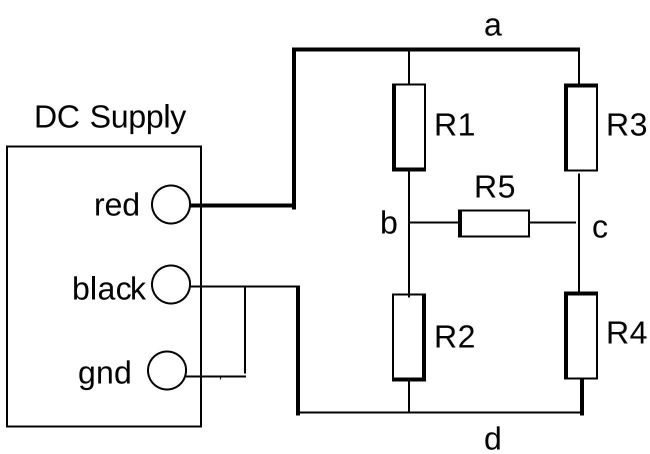

Figure 1. Wheatstone bridge circuit.

Notice that V_0 appears on both sides of eq.2 hence it cancels out. This has the important consequence that “errors” (noise, calibration, etc) in V_0 is irrelevant. A similar cancellation occurs for the resistors, in the following sense. We use a potentiometer for R1 and R2 as shown in figure 2. This circuit follows the voltage divider equation:

$$f = V_{bc}/V_0 = R_2/(R_1+R_2). \qquad \text{Eq. 3}$$

Here we have defined the ratio as a fraction “f” which ranges from 0 to 1. This is also aptly stated in words as "fraction of voltage equals fraction of resistance". In our experiment, the value of “f” is read directly from the potentiometer setting. Combining eqs 2 and 3 we have

$$f = R_4/(R_3+R_4) \qquad \text{Eq. 4}$$

Note that the values (hence errors!) for R1, R2 have disappeared. We can invert this to solve for the unknown R_4 as

$$R_4 = R_3\, f/(1-f) \qquad \text{Eq. 5}$$

Notice the astounding result that errors in R_4 are determined only by errors in R_3 and the potentiometer setting “f”. Given an accurate “standard” for R_3 and a good potentiometer, we can make extremely good measurements of R_4, even with a crummy sensing meter.

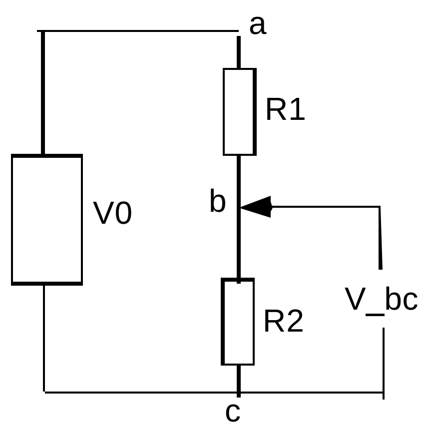

Fig. 2 Potentiometer shown as a voltage divider.

Kirchoff’s laws:

Recall Kirchoff’s laws, which represent conservation of energy and charge for an electrical circuit. These are

$$\Sigma V_j=0 \text{ around a closed loop} \qquad \text{Eq. 6}$$

$$\Sigma I_j = 0 \text{ into a node} \qquad \text{Eq. 7}$$

Procedure and analysis

1. Potentiometer: With no voltage applied (!), set the pot dial to f = 0.200, then measure R_1, R_2 and (R_1+R_2) using the ohms range of the DMM. The manufacturer says this is good to 0.5%. Your measurements may be worse! As usual take a few independent readings. Do your results fit Eq. 3?
2. Construct the Wheatstone bridge using resistor and voltage values as given in class. Review the earlier handout on "meters and wiring tips".
3. Adjust the potentiometer to balance the bridge. Errors in "f" should be determined from independent trials: spin the dial to unbalance the bridge then have each lab partner dial in a null and record this reading. Repeat several times.
4. Find the fractional uncertainty in R_4 (that is $\Delta R_4/R_4$) from Eq. 5, assuming R_3 is "perfect." Do this by analytic error propagation (calculus). Note that you don't need to know the values of R_3 and R_4 to do this.
5. Now we cheat and directly measure the resistor values. Disconnect the circuit and measure R_3 and R_4 using the DMM. Do these agree with your bridge measurement? You may neglect the uncertainty in "f" for this calculation. Put the circuit back together for the next measurements.
6. Unbalance the bridge by setting nominal f ~ 0.5. Measure the voltages around a closed loop moving CW from ground. That is V_{da} (across V_0) V_{ac} V_{cb} V_{bd}. Take care to maintain polarity as you move around! Compare with Kirchoff's law for voltage loop.
7. Node sum: Measure the currents into a single node, say point "a" of the circuit. Take care to maintain polarity for each value! Compare with Kirchoff's law for current node.