PHY 132 – Summer 2000
LAB 9: LRC Circuit (Phases)[1]

Introduction

In this lab we will measure the phases (voltage vs current) for each component in a series LRC circuit.

Theory

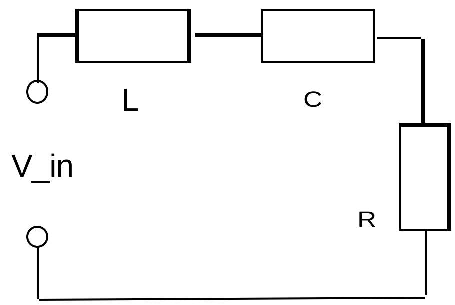

Fig. 1 Generic series LRC circuit.

Consider the steady-state (sinewave) behavior of a series LRC circuit as shown in Fig. 1. By definition, the same current flows through each component of a series circuit, and can be written as

$$i(t) = I_0 \sin(\omega t) \qquad \text{Eq. 1}$$

This waveform has amplitude I_0 and phase angle zero. The voltage across each component is a sinewave at the same frequency given by

$$v_j(t) = V_j \sin(\omega t + \Phi_j). \qquad \text{Eq. 2}$$

Note that in this context, v(t) is completely specified by amplitude and phase. Voltage amplitudes are given by Ohm's law (using AC reactances) as

$$V_j = I_0 X_j \qquad \text{Eq. 3}$$

and phase is given by

$$\tan(\Phi_j) = (X/R)_j \qquad \text{Eq. 4}$$

[1] Adapted by R. J. Jacob from P. Bennett, PHY-132 Lab Manual© (ASU)

Values for X and Φ are shown in table 1 below.

Component(s)	$X = \|Z\|$	Φ
R	R	0
C	$1/(wC)$	$-\pi/2$
L – ideal coil	wL	$+\pi/2$
Series (L+R+C)	$\sqrt{}(R^2+(X_L-X_C)^2)$	$\tan^{-1}(X_L-X_C)/(R)$
(L+r) - real coil	$\sqrt{}(r^2+(wL)^2)$	$\tan^{-1}(X_L/r)$

Table 1. Reactance and phase angle for various circuit components and combinations.

Phase shift can be determined directly from your scope trace by finding the time between successive zero crossings (Δt) compared to the full period of the waves (T). The phase shift is then simply given by the ratios

$$\Phi/2\pi = \Delta t/T \qquad \text{Eq. 5}$$

An example is shown in Fig. 2 for the case of $\omega = 2000$ rad/sec, Vout/Vin = $1/\sqrt{2}$ and $\Phi \sim 2\pi(0.4\text{msec}/3.14\text{msec}) \sim -\pi/4$ radians. Note that v_{out} is lagging v_{in} in accord with the negative phase shift.

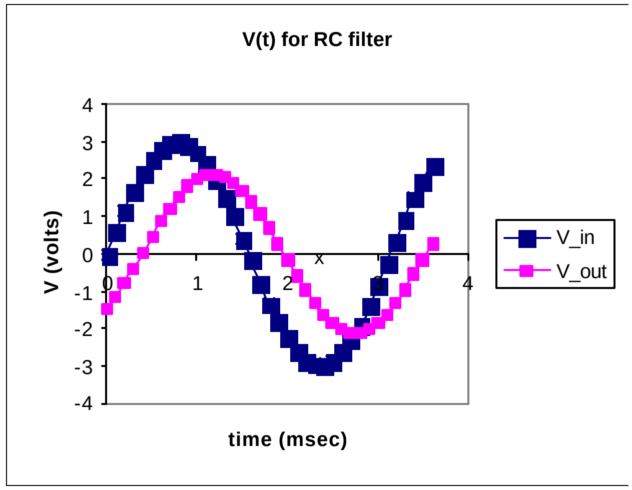

Fig. 2 Sample v(t) waveforms for RC filter.

Note that the circuit has a resonant frequency where X_L=X_C or $\omega_0 L$ = $1/(\omega_0 C)$ or

$$\omega_0^2 LC=1 \qquad \text{Eq. 6}$$

At this frequency, the total impedance is minimum, and given by Z_{LRC} (ω_0) = R. The current flow is maximized and the phase shift is zero. Resonance can usually be more accurately determined from phase shift than from amplitude, since a zero can be determined more accurately than a maximum.

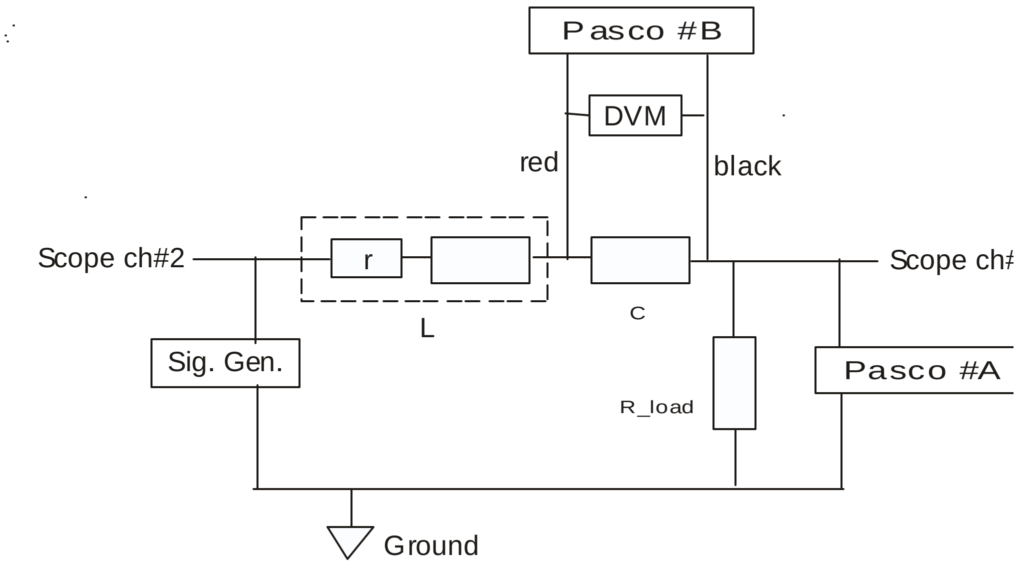

Fig. 3 Circuit used for measuring LRC behavior.

The circuit we will use is shown in Fig. 3. Note that the coil is non-superconducting and has an unavoidable non-zero internal resistance "r". This circuit element is only experimentally accessible as the combination (L+r). The internal "r" contributes to the total circuit resistance R in the equations and table above. Thus we have

$$R = R_{load} + r \qquad \text{Eq. 7}$$

Scope ch #2 senses the input voltage, while scope ch #1 senses the series current, given by voltage across the series resistor R_{load}. This voltage is in phase with the current because the resistor impedance has zero phase shift. We will also connect the Pasco voltage probes to various components in turn and "capture" the waveform. It is "triggered" on i(t), to guarantee that the captured wave is synchronized from one measurement to the next.

Procedure

1. Connect the circuit of Fig. 3 using R = 20 Ω, C = 10 μfd and L = 85 mH.
2. Input a sinewave about 100 Hz, $2V_{pp}$ (peak-to-peak on scope ch #2). Get both waves showing on the scope simultaneously. Find and record the resonant

frequency f_0 where the current (scope ch. #1) is maximum and phase shift is zero.

3. Sketch and describe the waveforms as you tune through resonance. How can you tell that the phase changes sign?
4. Set the frequency a little below resonance, where the output voltage is about ½ of its value at resonance. Does the voltage across the LRC circuit (scope ch #2) lead or lag the current (ch #1) under these conditions?
5. Using the DVM, measure voltages across each component around the circuit, including v_{in}.
6. Next, digitize the waveform v(t) for each component in the circuit. Load the setup file "LRCphase.sws". Connect the Pasco probe B to each component in turn around the circuit and capture the waveform. Be sure that probe A triggers properly, giving a sinewave with zero phase shift. Be careful to observe the current polarity when you connect the Pasco probes, using black on the clockwise side of each component, including the signal generator. Note that the scope cannot be moved around the circuit because one side is necessarily grounded, but the Pasco probe has "floating" inputs, so they can be connected anywhere. Copy/paste or export this data to EXCEL or GA for analysis.

Analysis

1. Compare your resonant frequency f_0 with theory. Component values are accurate to R(1%), C(10%) and L(5%).
2. Plot the waveforms for each component (Pasco-B) including the signal generator. You should find that the i(t) waveforms (Pasco-A) match for all components if triggering was done correctly. Find the voltage phase angle and

amplitude for each component. This is illustrated in figure 2 above. Present these in a single table showing phase angle and voltage amplitude ratio V_j/V_{in}.

3. Add the V_{rms} voltages around the circuit (v_R, v_L, $v_{C)}$ and compare with v_{in}. Why do these values "not follow Kirchhoff's law"?
4. Plot all $v_j(t)$ waveforms (v_R, v_L, v_C and v_{in}) and their sum on a single chart. Do these waveforms follow Kirchhoff's law?