PHY 122 LAB 10: COLLISIONS IN TWO DIMENSIONS

INTRODUCTION

In this lab we want to learn about conservation laws by studying collisions between two bodies. You will be provided with an air table and round pucks which float on a cushion of air. You will follow the movement of the pucks on a TV monitor. Transparencies will be provided to record the positions of the pucks as a function of time. Once a desirable collision is obtained, you will trace the paths of the pucks on the monitor screen when the event is played back from the VCR in slow motion.

The analysis of a collision requires the use of the concept of momentum. Since you may not be familiar with this concept, we will discuss it briefly.

MOMENTUM

Momentum $\mathbf{p}$ is defined as

$$\mathbf{p} = m\mathbf{v}.$$

where m is the mass of the particle and $\mathbf{v}$ is its velocity. Momentum is a vector. In terms of momentum, the net force on a particle can be defined by

$$\Sigma\mathbf{F} = \frac{d}{dt}\mathbf{p}.$$

If the mass of the particle is constant, this gives

$$\Sigma\mathbf{F} = \frac{d}{dt}\mathbf{p} = \frac{d}{dt}(m\mathbf{v}) = m\mathbf{a}$$

which is Newton's Second Law.

If two particles collide or interact, Newton's Third Law states $\mathbf{F}_{12}$=-$\mathbf{F}_{21}$. In terms of momentum

$$\frac{d}{dt}\mathbf{p}_1 = \mathbf{F}_{12} = -\mathbf{F}_{21} = -\frac{d}{dt}\mathbf{p}_2,$$

so that the total momentum $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$ satisfies

$$\frac{d}{dt}\mathbf{P} = \frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2) = \frac{d}{dt}\mathbf{p}_1 + \frac{d}{dt}\mathbf{p}_2 = \frac{d}{dt}\mathbf{p}_1 - \frac{d}{dt}\mathbf{p}_1 = 0.$$

This means that the total momemtum is a constant as a function of time. In other words, $\mathbf{P}$ is conserved. This is only true, however, IF there are no external forces acting on the system of particles.

COLLISIONS

In a collision, two bodies interact in such a way that their states of motion are well defined before and after the event. In the simplest case, no external forces act upon the bodies, so that Conservation of Momentum implies

$$\mathbf{P}_{final} = \mathbf{p}_{1,f} + \mathbf{p}_{2,f} = \mathbf{P}_{initial} = \mathbf{p}_{1,i} + \mathbf{p}_{2,i}.$$

Since momentum is a vector, the above equation is a compact notation for three conservation equations, one for each of the three components P_x, P_y, and P_z.

A collision in which kinetic energy is constant is called elastic. Thus in an elastic collision,

$$K_{final} = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2 = \frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = K_{initial}.$$

Conversely, in an inelastic collision

$$K_{final} \neq K_{initial}.$$

In all collisions, the total energy is conserved if there are no external forces. Total energy, however, is usually difficult to measure, because it can be stored in the form of heat or potential energy, as in the case of a deformed body.

EXPERIMENT

You will perform two experiments. The first one should be somewhat elastic, while the second one will be a perfectly inelastic collision. You will analyze your data to determine the fraction of kinetic energy lost for each collision.

In both cases, measure the initial and final momenta and kinetic energies and discuss your results in terms of the conservation principles for momentum and energy. In order to make the analysis easier, puck 2 should be at rest before the collision, so that $\mathbf{p}_{2,i} = 0$ and $\frac{1}{2}m_2 v_{2,i}^2 = 0$.

Remember that in order to claim two quantities are equal you have to show that there is some overlap between the range of values defined by their best values plus or minus their uncertainties.

For the measurement of momentum, a vector quantity, you need to consider two dimensions. YOU SHOULD VERIFY MOMENTUM CONSERVATION IN THE x DIRECTION

AND IN THE y DIRECTION. You can define these directions however seems best to you. You will probably conclude that the most convenient way to define your coordinate system is to choose the x axis along the line that joins puck 1 with puck 2 before the collision. But you are free to make other choices.

A complication arises in the computation of kinetic energy. Our pucks are not particles, and can rotate as well as translate. This rotation involves some kinetic energy. The rotational kinetic energy of the puck is given by

$$K_{rot} = \frac{1}{2} I\omega^2$$

where I is the moment of inertia, given in the case of the puck by $I = \frac{1}{2}MR^2$, where M is the mass of the puck and R is its radius. The quantity ω is the angular velocity of the puck in radians/second (1 revolution/second = 2π radians/second). So the total kinetic energy of each puck is its translational kinetic energy $\frac{1}{2}MV^2$ PLUS its rotational kinetic energy $\frac{1}{2}I\omega^2$. If the rotational kinetic energy of a puck is very small, less than the uncertainty in the translational kinetic energy, then you can neglect it.

For the perfectly inelastic collision, the two pucks will stick together after the collision. In this case, the two pucks will rotate about their common center of mass (the point where they are stuck together). The moment of inertia of the two pucks about this point is $3MR^2$, where M and R are again the mass and radius of an individual puck.

If you compare initial and final kinetic energies and they happen to be the same within experimental error, you can say that the collision is elastic. Be aware, however, that your TA has a method to quickly make sure that your statement is correct. If the two pucks have equal masses, they will move off at 90° to each other if the collision if perfectly elastic. Can you show that this is true?

To establish errors in puck speed, you should tabulate the speed for many different intervals. You may estimate your error in the angle of the puck's motion with respect to the selected x axis. For the angular velocity (spin) of each puck about its center you may simply use the initial and final angles over the total intervals that you considered in your velocity calculation. You may consider puck radius, time, and spin angle to be exact. If you arrange for both pucks to be of equal mass, then the mass will divide out of your calculations (and for the kinetic energy the $\frac{1}{2}$ will also divide out).

For each collision, tabulate the following quantities for each puck in a single table (one table for before the collision and one table for after): speed, angle wrt the x axis, v_x, v_y, v^2, ω, $\frac{1}{2}R^2\omega^2$, and $v^2 + \frac{1}{2}R^2\omega^2$. Why is this last quantity relevant? Include uncertainties in the tables where appropriate (for all quantities except ω, and $\frac{1}{2}R^2\omega^2$). In the perfectly inelastic collsion, after the collision the two pucks are stuck together; so there is only one object moving. In this case, you will probably want to change several of your table headings. For example, for most convenience, the last two table headings should probably be $3R^2\omega^2$, and $2v^2 + 3R^2\omega^2$. Again, why?

For each collsion, state your conclusions clearly. Is momentum conserved for each component? What is the fractional change in total K?

THINGS TO THINK ABOUT

How are you going to measure distances from the transparency? Does it matter that the distances measured do not give the distances on the air table directly? What about the radius R of the puck?

Why is it critical that the air table not be tilted?