SIMPLE HARMONIC MOTION

Summer 2001

Simple Harmonic Motion (SHM) is periodic sinusoidal motion in one dimension. Periodic means that the motion is repetitive, and sinusoidal means that the graph of x vs. t looks like a sine funtion. The simplest system is a weight suspended by a spring. After achieving equilibrium with respect to the gravitational force (which then becomes effectively irrelevant to the subsequent motion of the weight), the weight will remain stationary until it is displaced either up or down. Then the weight will perform SHM about its position of equilibrium.

If the spring is "massless", the frequency of the SHM is given by $f = \frac{\sqrt{k/m}}{2\pi}$ where k is the spring constant and m is the mass of the weight. The springs we will use are specially designed so that one can easily correct for their mass m_s by adding one-third of it to the mass of the weight. Thus $f = \frac{\sqrt{k/M}}{2\pi}$ where $M = m + (m_s/3)$.

If the SHM is impeded by friction or air resistance it will lose energy, the vibrations will become progressively smaller, and the SHM will eventually stop. This is called *damped* motion. In such a case the frequency is slightly altered to $f = \sqrt{f_0^2 - (\frac{b}{4\pi M})^2}$ where f_0 is the undamped frequency and b is the ratio of the magnitude of the damping force to the speed of the weight.

Part One: DETERMINATION OF k.

You will have a set of three 10-g masses and a mass hanger. Use these to determine the spring constant of your spring. Use the motion sensor and Science Workshop software to determine the position of the hanging mass, and use Graphical Analysis to make your graph of force vs. position.

Part Two: SIMPLE HARMONIC MOTION

Now use the piece of brass with the small reflecting screen as a hanging weight. Measure the mass of this weight. Also measure the mass of the spring and determine the theoretical undamped frequency of the system. Then set the weight into motion with a moderate amplitude. Use the motion sensor and Science Workshop to plot the position x of the mass vs. time t for at least five complete oscillations of the mass. For a complete oscillation the mass has to be back in the same place moving in the same direction.

Does the motion of the mass appear to be sinusoidal? Check this in Graphical Analysis by first computing $\Delta x = x - \bar{x}$ where $\bar{x}$ represents the equilibrium position of the SHM. Then fit the graph of Δx vs t with a Sin function of the form $A * sin(Bx + C)$. Hints: The fit will proceed much faster if you enter the approximate value of B by hand -- B is the angular frequency or $2\pi f$. Also, your fit should be nearly perfect if you add a decay factor; i.e. fit to $A * e^{-Dx} * sin(Bx + C)$. The decay factor will allow for a gradual decrease in amplitude of the SHM.

Determine the frequency of the motion directly from the graph and also from the fit. Are these values within the uncertainty of your calculated value of f from the equations above? Discuss possible reasons for any difference.

Part Three: CONSERVATION OF ENERGY

The potential energy U of your spring is $\frac{1}{2}k\Delta x^2$. The kinetic energy K of the moving mass plus spring is $\frac{1}{2}Mv^2$ where M is the corrected mass from above. We would like to find K + U = E versus t. We already have $\Delta x(t)$. To create a velocity column use the derivative function of Graphical Analysis to get $v(t)$ from $x(t)$. Now calculate extra columns for $U(t)$, $K(t)$, and $E(t)$, and plot each versus time. It is instructive to use the same energy and time scaling for all three graphs, and then to arrange the three graphs together on one page with the U vs. t graph at the top, the K vs. t graph in the middle, and the E versus t graph at the bottom. Use a regression line and statistics for $E(t)$. To what degree is energy conserved over the time during which you took data? What has happened to any energy that has been "lost"?

USING SCIENCE WORKSHOP

1. Double-click on the icon PHY122-SHM to start the program.
2. To start data collection click on REC. Data collection stops automatically after six seconds.
3. For Part One transfer your data to Graphical Analysis by hand, since you only have a few data points. For Part Two use Copy and Paste (under Edit) to transfer your data to Graphical Analysis.
4. IMPORTANT. When you EXIT the program DO NOT save any changes.