FARADAY'S LAW*

Purpose of the Experiment

In this experiment, you will observe induced EMF's in a solenoid and compare their time dependences with those predicted by Faraday's Law.

Introduction to the Physics

Because the material relevant to this experiment will not be covered in the lecture course until the middle of the term, it is important here to present at least an operational discussion of the underlying physics. The student is also referred to Sections 29.1-3 in the textbook by Young and Freedman (12th edition).

Ampere's Law and Solenoids

The beginnings of modern electrical science and technology can be traced to two very important discoveries in the early Nineteenth Century. The first, seen first in many different contexts by several scientists, but generally called **Ampere's Law**, recognizes that electrical currents give rise to magnetic fields and gives the mathematical relationship between them.

Thus, for example, using Ampere's Law (and Gauss' Theorem for magnetic fields, $\oint \vec{B} \cdot \vec{dA} = 0$) one can deduce expressions for the magnetic fields associated with a conducting solenoid. The solenoid used in this experiment is a long, tightly wound, circular coil of insulated wire. If the solenoid has N_1 loops, or "turns" over a total length, L_1, then when it is conducting a current I_1, the interior magnetic field is directed along the solenoid's axis and has the magnitude

$$B_1 = \frac{\mu_0 N_1 I_1}{L_1},\tag{1}$$

where μ_0 is the permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.

Faraday's Law

The second great discovery relating electrical and magnetic phenomena was the work of Faraday, Henry and Lenz. Its mathematical expression is called **Faraday's Law**. Qualitatively, Faraday's Law says that time-varying magnetic fields induce electromotive forces, or non-electrostatic voltage drops, that can, among other things, cause currents to flow. (These currents will themselves generate additional magnetic fields which, according

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to Lenz's Law, will in general oppose the changes in the original magnetic fields, but this effect will have very little bearing on your particular experimental results.)

We can generate a time-dependent magnetic field, $B_1(t)$ inside our solenoid by varying the current. B_1's time dependence will be very nearly the same as $I_1(t)$'s. Then, according to Faraday's Law, a small wire probe placed in the solenoid should experience an induced EMF. Indeed, if the probe is itself a coil of N_2 turns and effective loop area A_2, and if its axis is parallel to the solenoid's, Faraday's Law gives us the quantitative expression for the induced EMF within the probe coil:

$$\mathcal{E}_{ind} = -N_2 A_2 \frac{dB_1}{dt}. \tag{2}$$

The right-hand side of this expression is just the negative of the time derivative of the total magnetic flux, Φ_2, linking the probe.

The induced EMF in the probe will generate a voltage signal that can be displayed, along with the original signal voltage, on an oscilloscope.

Apparatus

Your team is provided with the following apparatus:

1. A signal generator that can provide three wave forms: square wave, triangular and sinusoidal.
2. An oscilloscope.
3. A large solenoid with $N_1 = 550$ turns.
4. A probe coil with $N_2 = 315$ turns.
5. A resistance box and various connection cables. The resistance box should be set at a value far exceeding the resistance of the solenoid.
6. Rulers and micrometers.

The solenoid should be connected through the resistance box and the signal generator as shown in the diagram on the following page. Connect both the solenoid and the probe to the oscilloscope as indicated. Please note that the probe and the solenoid connectors are fragile. Handle the equipment with care.
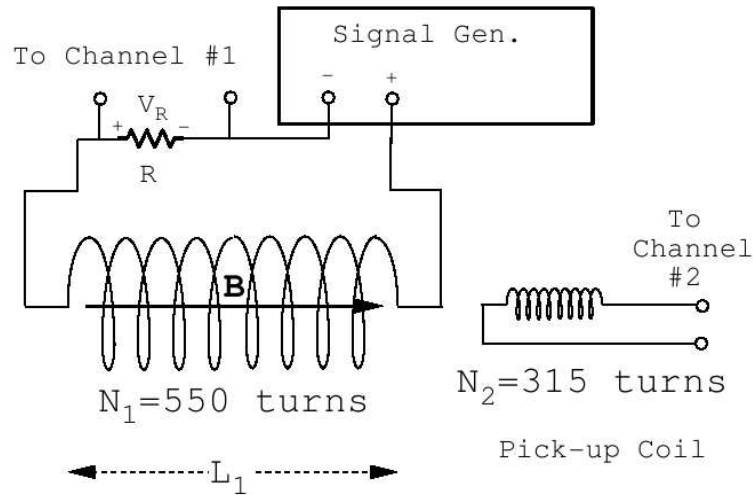
Familiarize yourselves with the apparatus by observing the effects of adjusting frequency, amplitude and resistance controls on the signal generator's and probe's waveform displays.

Experimental Plan

You have the possibility of testing Faraday's Law for three different periodic time-dependencies of inducing fields. The parameters you can adjust are the amplitude and the frequency; the geometry and numbers of turns in the coils are fixed.

You should begin by considering in turn each of the input signal wave forms at your disposal and attempting to predict the behavior of the output signal from the probe with respect to the parameters you can vary. Thus, for example, how do you expect the time-dependence of the output signal to be related to that of the input? How should the amplitude of the output depend upon the amplitude and, or, the frequency of the input? Should the amplitude of the input affect the frequency of the output? In your analyses (as long as R is much larger than the internal impedence of the solenoid), you may use the relationship $I_1 = V_{sg}/R$ where R is the resistance box setting and V_{sg} is the signal generator voltage.

After having discussed these issues as a team, procede to develop a plan of measurements that would test your conclusions. Do this for each of the input wave-forms and produce tables and graphs that display your data and illustrate your results. Be sure to estimate the uncertainties in your measurements.

Hints: A Sample Experimental Procedure

If your team is having difficulty getting started, this sample procedure is provided for the sinusoidal wave-form.

We switch the signal generator wave-form selector to "sinusoidal" and suppose that the voltage signal picked up across the resistor is

$$V_r = V_{r,m} \sin(2\pi f t),$$

where $V_{r,m}$ is the amplitude and f is the frequency. Both can be measured on the oscilloscope screen if we adjust the amplification and sweep to appropriate values.

With this expression for $V_r(t)$, we have

$$I_1(t) = \frac{V_{r,m}}{R} \sin(2\pi f t).$$

Then, from equation 1,
$$B_1 = \frac{\mu_0 N_1}{L_1} \frac{V_{r,m}}{R} \sin(2\pi f t).$$

To find the induced emf, we take the time-derivative of this and insert it into equation 2 to obtain
$$\mathcal{E}_{ind} = -\frac{2\pi\mu_0 f}{L_1 R} N_1 N_2 A_2 V_{r,m} \cos(2\pi f t). \tag{3}$$

This is the equation of the wave form we should be observing as the lower trace on the screen. We note three important features: (1) The induced EMF is proportional to the signal voltage. (2) The induced EMF is proportional to the driving frequency. (3) The signal voltage does not affect the output frequency, but is 90° out of phase with the induced EMF. All of these statements can be checked quantitatively by adjusting the input frequency and voltage and measuring the output. In particular, a plot of $\mathcal{E}_{ind,m}$ (which is the amplitude of the induced EMF in equation 3) vs. f should be a straight line with slope equal to $2\pi\mu_0 N_1 N_2 A_2 V_{r,m}/(L_1 R)$.

Since we know, or can measure, all the other parameters in equation 3, we can then compare our observed results with theory.