Consider a robot that is moving in a square environment with four areas of interest denoted by $\pi_1, \pi_2, \pi_3$, and $\pi_4$. Initially, the robot is placed somewhere in the region labeled by $\pi_1$. The desired specification for the robot given in natural language is: "Visit area $\pi_1$, then area $\pi_2$, then area $\pi_3$, and eventually return to region $\pi_1$ while avoiding areas $\pi_2$ and $\pi_3$.

LTL specifications

The formulas are built from a finite number of atomic propositions $\Pi$ which label areas of interest in the environment such as rooms or obstacles. Proposition $\pi_i \in \Pi$ represents an area of interest in the environment which can be characterized by a convex set of the form:

$$ P_i = \{ x \in \mathbb{R}^2 \mid a_1 x + b_1 \leq 0, a_i x + b_i \leq 0, a_4 x + b_4 \leq 0, a_4 x + b_4 \geq 0 \} $$

The propositional formulas are formed using the traditional operators of conjunction ($\land$), disjunction ($\lor$), negation ($\neg$), implication ($\Rightarrow$), and equivalence ($\Leftrightarrow$. LTL-$\mathcal{X}$ formulas are obtained from the standard propositional logic by adding temporal operators such as eventually ($\Diamond$), always ($\Box$), and until ($U$).

Some LTL examples that express interesting properties in the context of mobile robot motion planning include:

- Reachability: The formula $"(a_1 v a_2 \ldots v a_6) U m"$ expresses the property that eventually $m$ will be true, and until $m$ is reached, we must avoid obstacles labeled as $a_1, a_2, \ldots, a_6$.
- Reversal: The requirement that we must visit $\pi_1$, $\pi_2$, and $\pi_3$ in that order is naturally captured by the formula $"(a_1 o (a_2 \lor a_3))"$.
- Coverage: Formula $"(a_1 \land (a_2 \lor a_3))"$ reads on the robot will eventually reach $\pi_1$ and eventually $\pi_2$ and eventually $\pi_3$ required the robot to eventually visit all regions of interest.
- Recurrence: The formula $"(a_1 \land a_2 \land a_3 \land U m)"$ requires that the robot trajectory does whatever the coverage does and, in addition, will force the robot to repeat the desired objective infinitely.

Temporal Logic Motion Planning for Mobile Robots

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Overview of Temporal Logic Motion Planning

Input: Specification
In Natural Language
Input: Polygonal Environment $P$

Linear Temporal Logic

Triangulation & Finite Transition Sys.

Open-Loop Hybrid Controller

Close-Loop Hybrid Controller

Hybrid Controller Implementation

Continuous Model

A triangulation is a bi-simulation if the system can move between any two adjacent triangles regardless of the initial state. For each triangle, we design three controllers ensuring that system exits the triangle from the desired facet to the adjacent triangle.

Thm: There exist (many) affine vector fields $\nu : \Pi \times \mathbb{R}^2 \rightarrow \mathbb{R}^2$ on any triangle, satisfying the bisimulation property.

Then, there exist (many) affine vector fields $\nu : \Pi \times \mathbb{R}^2 \rightarrow \mathbb{R}^2$ on any triangle, satisfying the bisimulation property.

Affine functions on simplexes are uniquely defined on vertices. The set of all controllers can be parameterized by the values on the vertices.

Spec: Go to area 1, 2, 3, 4, 5, 6 in no particular order

Spec: Go to the two black rooms

Spec: Go to area 2, then to area 1 and then cover areas 3, 4, 5 - all this, while avoiding obstacles $\pi_2, \pi_3, \pi_4$

Spec: Visit all highlighted areas

Examples

Problem Size: 1156 observables, 9250 triangles, Solution path: 5 seconds, Matlab: 90 seconds

Computation Time: 1 second, NuSMV: 55 seconds, Matlab: 90 seconds

The trajectory generated by NuSMV, satisfying this formula is: 33, 34, 24, 25, 27, 16, 15, 14, 3, 4, 5, 32, 26, 29, 30, 3, 14, 33

An example of a robot trajectory generated by the NuSMV model checker for the given LTL formula.

Problem Formulation

Model: We consider a fully actuated, planar model of robot motion operating in a polygonal environment $P$. The motion of the robot is expressed as:

$$ \dot{x}(t) = u(t), \quad x(t) \in P \subseteq \mathbb{R}^2, \quad u(t) \in U \subseteq \mathbb{R}^2 $$

Specification: A linear temporal logic (LTL) formula $\phi$ that captures the robots' desired behavior.

Problem: Given robot model, environment $P$, initial condition $x(0)$, and an LTL-$\mathcal{X}$ temporal logic formula $\phi$, find control input $u(t)$ such that $x(t)$ satisfies $\phi$. For instance, $\phi = \Box (\pi_2 \land \pi_3)$ may mean that the trajectory of the mobile robot starts at $\pi_2$ and eventually reaches $\pi_3$.