On an Optimal Algorithm for Channel Assignment in Cellular Networks

Arunabha Sen and Tom Roxborough
Department of Computer Science and Engineering
Arizona State University
Tempe, AZ 85287

Bhabani P. Sinha
School of Computer Science
University of Central Florida
Orlando, FL 32816

Abstract

A cellular network is often modelled as a graph and the channel assignment problem is formulated as a coloring problem of the graph. In a recent paper [16], we introduced the notion of *cellular graphs* that models the hexagonal cell structure of a cellular network. Assuming a *k-band buffering* system where the interference does not extend beyond k cells away from the call originating cell, we provided two different formulations of the channel assignment problem - *distance-k chromatic number* problem and *k-band chromatic bandwidth* problem. The channel assignment algorithms presented in [16] were *non-optimal*. In this paper we provide (i) a new algorithm for the *distance-k chromatic number* problem that is *optimal* and (ii) a near optimal algorithm for *2-band chromatic bandwidth* problem that has a performance bound of 4/3. The complexity of the algorithms is $O(p)$, where p is the number of cells.

1 Introduction

In a mobile cellular network the service area is divided into a number of cells, which are typically represented by a regular hexagonal grid as shown in figure 1. Each cell is allocated a set of frequency channels to meet the traffic demand in that cell. The same frequency channel can be used in two different cells as long as there is no perceptible interference. Although the geographical distance between the cells is not the only factor in determining the interference between the cells, it is a major factor. If the distance between two different cells in the service area is sufficiently large, then there will be no interference even when the same frequency channel is used for communication in these two cells. Such cells are known as *co-channel* cells. There may not be interference between the cells even when the distance between the cells is not sufficiently large, due to the presence of large buildings or structures in the service area. In this paper we optimize the channel use under the assumption that the interference between the cells is determined only by the geographical separation between the cells. Thus the results presented in this paper may be considered as the upper bounds for the situations where the interference is influenced by factors other than the geographical location.

A cellular network is often modelled as a graph and the channel assignment problem is formulated as a coloring problem of that graph. The channel assignment problem in its most general form is an NP-complete problem, as the chromatic number problem of a graph is a special case of the channel assignment problem. In one of the earliest papers on the channel assignment problem, Box [1] proposed a simple procedure based on the ranking of the channel requirement of calls in descending order of their *assignment difficulty*. Hale in [6] presented a fairly exhaustive study of the different versions of frequency assignment problems encountered in television and radio environments. Gamst in [3] developed a theory of optimal adjacent channel distance for a homogeneous hexagonal cell system. Some of the channel assignment algorithms in his paper were also discussed in Macdonald's seminal paper [12] on cellular concepts. In [4] Gamst presented some lower bounds for a class of frequency assignment problems.

More recently, simulated annealing [2, 13] and neural networks [10] have been used in solving the channel assignment problem. Sivarajan [18] studied the problem as a generalization of the graph coloring problem. Based on the notion of *channel reuse*, Kim and Kim [9] presented a two phase algorithm for the channel assignment problem. Sengoku *et. al.* [17] studied a version of the problem known as the *channel offset scheme* from a graph theoretic standpoint. Using the same formulation as [18], Wang and Rusforth [20] present an adaptive local search algorithm for the graph coloring problem. Tcha *et. al* [19] very recently presented some new results on the lower bound for the frequency assignment problem thus improving on the earlier results by Gamst.

In all these studies the graph used to model the cellular network ignores the geometry of the network. However, the cellular network has a very regular structure. Only very recently, [8, 11, 14, 16], researchers have started taking notice of the fact that if the geometry of the network is taken into account then the graph modeling a channel assignment problem will have a very regular structure and exploiting this regular structure, the channel assignment problem can be solved optimally in some cases. In [16] we presented three channel assignment algorithms. The first algorithm considers only *co- channel* constraint and the other two consider both the co-channel and the *adjacent* channel constraints. We formulated the channel assignment problem with only co-channel constraint as the *distance-k chromatic number* problem. The channel assignment problem with both co-channel constraint and adjacent channel constraint is formulated as the *k-band chromatic bandwidth* problem. Unfortunately, the algorithms presented in [16] were not *optimal*. In this paper we provide a new algorithm for the *distance-k chromatic number* problem in a cellular graph that is *optimal*. For the *2-band chromatic bandwidth* problem, we present a *near optimal* algorithm with a performance bound of 4/3. Both the algorithms assign channels to the cells and the complexity of both of them is $O(p)$, where p is the number of cells.

2 Modeling the Cellular Network

Sengoku *et. al.* in [17] model the cellular network as a graph coloring problem, except that the graph constructed is somewhat different from the graph constructed in [9, 18, 20]. In this model each node of the graph represents a hexagonal cell of the network and two nodes have an edge between them if the geographic location of the cells are close enough so that the calls in one cell may interfere with the calls in the other cell. There are weights associated with the nodes as well

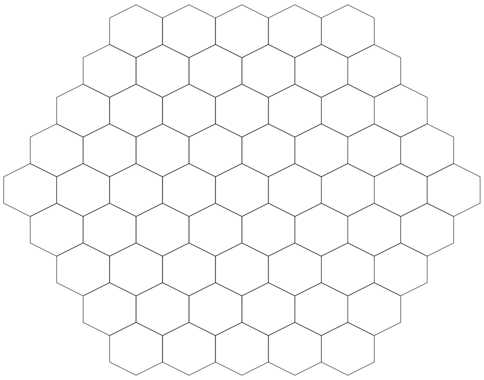

Figure 1. Hexagonal cell structure of cellular network

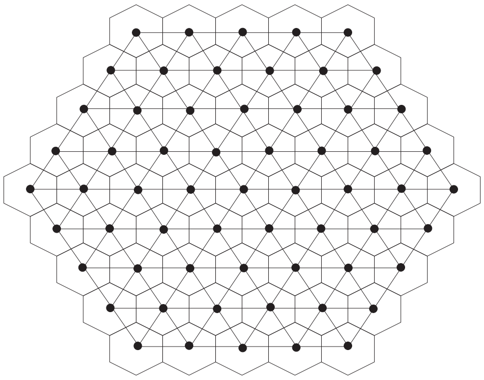

Figure 3. Cellular graph corresponding to the cellular network of Figure 1

as the edges. The weight $w1(i)$ associated with node i represents the number of calls in cell i and the weight $w2(i, j)$ associated with edge (i, j) represents the amount of interference between the cells i and j and hence specifies the minimum amount of frequency separation required between these two cells. This weight is comparable to the entries of the frequency separation matrix considered in [9, 18, 20]. This type of graph is referred to as the $interference\ graph$ of the cellular network [17].

Due to the nature of the radio transmission, calls generated in a certain cell i may interfere with the calls generated in a different cell j, only if the geographical separation distance between the cells is less than some threshold value, determined by the power used for the radio transmission. As such the models considered in both [9, 17] assume that two cells do not have any interference if their geographical separation distance is greater than a specified threshold value. The cellular network is said to have a *k-band buffering* system if it is assumed that the interference does not extend beyond k cells away from the cell where the call originates. An example of a buffering system with $k = 2$ is shown in figure 1. In the figure, Z_i represent the cell i and the letter A indicates that the same frequency A can be used in all these cells without interference.

In the model proposed in [16] the channel assignment problem was formulated as a generalization of the graph coloring problem. However, the graph constructed in that model is different from the models considered in [9, 18, 20] as well as in [17]. In this model, each cell of

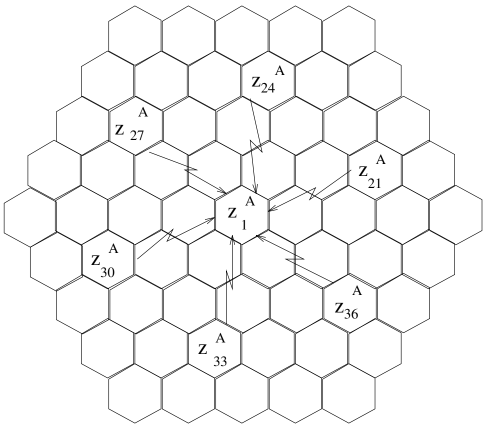

Figure 2. Two band buffering

the network is represented as a node and two nodes have an edge between them if the corresponding cells are adjacent to each other (i.e., the hexagonal cell boundaries share a common segment). This is basically the $adjacency\ graph$ of the cellular network and we will refer to this graph as the $cellular\ graph$. An example of the cellular graph generated from the cellular network of figure 2 is shown in figure 3. In this new model the cellular network is described by the following four components:

1. A cellular graph $G = (V, E)$.
2. A weight w_i, associated with node i of the graph G, representing the demand in node i of G (cell i of the network).
3. An integer value k that indicates the extent of interference beyond the call originating cell. In a *k-band* buffering system, interference does not extend beyond k cells away from the call originating cell.
4. A set of integer values $s_0, \ldots, s_k$ ($s_0 \geq s_1 \geq \ldots \geq s_k$) representing the frequency separation requirement between various nodes (cells). If the distance $d(i, j)$ between two nodes i and j in G is r, $0 \leq r \leq k$, then the assigned frequencies in the corresponding cells should be separated by at least s_r. If $r > k$ then these two cells are allowed to have the same frequency.

3 Channel Assignment in the Cellular Network

In cellular networks, two different kinds of interferences - *co-channel interference* and *adjacent channel interference* need to be taken into account while assigning channels to the cells. Co-channel interference takes place if the same frequency channel is assigned to two different cells which are geographically close so that a call in one cell may interfere with a call in the other cell. Adjacent channel interference takes place if the frequencies of channels assigned to two different cells are close on the frequency spectrum for the physical distance between the cells. In [16] we presented two different formulations of the channel assignment problem - one takes into account only the co-channel interference and the other considers both the co-channel and the adjacent channel interferences. We formalize the problems as *distance-k chromatic number problem* and *chromatic bandwidth problem* respectively.

Distance-k Chromatic Number Problem: Given a graph $G = (V, E)$, and an integer k, the distance-k chromatic number of the graph is the fewest number of colors needed to color the nodes of the graph so that no two nodes of the graph have the same color if the shortest path length between the nodes is less than or equal to k. Such a coloring of the nodes of the graph is known as a *proper* coloring.

Chromatic Bandwidth Problem: Given an edge weighted graph $G = (V, E)$ with weight $w(i, j)$ associated with the edge (i, j), the function $f(v_i), v_i \in V$ is a mapping (or coloring) of the set of nodes to the set of natural numbers, $\{f : v_i \rightarrow \mathcal{N}, v_i \in V\}$. The coloring is *proper* if for all $i, j \in V$, $| f(v_i) - f(v_j) | \geq w(i, j)$. ($w(i, j) = 0$ if $(i, j) \notin E$.) The *bandwidth* of a proper coloring (mapping) is the largest integer used in that mapping. The *chromatic bandwidth* of an edge weighted graph is the smallest bandwidth over all proper colorings of the graph.

The distance-k chromatic number problem corresponds to the channel assignment problem in a k-band buffering system where the interference extends up to k cells of the call originating cell and as such these cells cannot use the same channels as the ones in the call originating cell.

The chromatic bandwidth problem correspondences to the channel assignment problem when both the co-channel and the adjacent channel interference are taken into account. The weight $w(i, j)$ associated with the edge (i, j) in this problem represents the frequency separation requirement between the calls represented by the nodes i and j.

It may be noted that both the chromatic bandwidth problem and the distance-k chromatic number problem are defined in terms of any arbitrary graph $G = (V, E)$. However, if $G = (V, E)$ is an arbitrary graph it is easy to see that both the problems are NP-complete. This is true because the special case of the chromatic bandwidth problem where all edge weights are equal to 1 and the special case of distance-k chromatic number problem where $k = 1$ is the well known chromatic number problem, which is NP-complete [5]. However, for the frequency assignment problem, we restrict our attention only to *cellular graphs*. Consider a cellular network with a k-band buffering system. If $d(i, j)$ is the shortest path length between the nodes i and j in the graph, in the revised model of the channel assignment problem the edge weight $w(i, j) = s_{d(i,j)}$, for $0 \leq d(i, j) \leq k$ and zero otherwise (s_r is the frequency separation requirement between two nodes that are distance r apart in the cellular graph).

The physical interpretation of the k-band chromatic bandwidth problem is that if the distance between two cells is more than k, they are allowed to use the same frequency channel. The frequency separation requirement varies inversely with the distance between the cells. This has a very natural correspondence with the cellular networks where the interference between two calls is maximum when the calls are in adjacent cells and the interference decreases as the distance between the cells increases thus permitting a smaller amount of frequency separation.

4 Special Cases in the Model

Now we consider some special cases of the channel assignment problem in a cellular network. First we restrict our attention to cellular network with *homogeneous traffic*, i.e., demand in all the cells of the network is almost identical. Suppose that the number of calls in each cell is W (i.e., $w_i = W$ for all i) and the frequency separation requirement between calls in the same cell is s_0. Suppose the minimum bandwidth necessary for the situation when W = 1, satisfying the frequency separation requirement ($s_0 \geq s_1 \geq \ldots \geq s_k$) is B. Then for the situation when $W > 1$, the minimum bandwidth that will be needed is given by $max(s_0, B)W$. Therefore, for the homogeneous traffic ($w_i = W$ for all i), we analyze the assignment problem with the assumption $W = 1$, as the bandwidth for $W > 1$ can easily be obtained by multiplying the bandwidth obtained for $W = 1$ by the factor $max(s_0, B)$. Although the homogeneous traffic assumption is somewhat restrictive, these results are not only directly applicable to networks where the assumption is valid, they also are useful in reducing the problem size for a *non-homogeneous* network as is shown in [20].

Case I: Frequency separation s_r = 1 for all r, $0 \leq r \leq k$.

It is clear that with the interference band k being arbitrary and frequency separation s_r = 1 for all r, $0 \leq r \leq k$, the frequency assignment problem is equivalent to the computation of the *distance-k chromatic number* of cellular graphs, which we describe next.

Coloring Scheme for Distance-k Coloring of Nodes in Cellular Graphs

In this section we present the scheme to distance-k color a cellular graph with $3/4(k + 1)^2 + 1/4$ colors if k is even and $3/4(k + 1)^2$ colors if k is odd. The following theorem was proved in [16].

Theorem 1 *The distance-k chromatic number of cellular graphs is at least* $3/4(k + 1)^2$ *if k is odd and is at least* $3/4(k + 1)^2 + 1/4$ *if k is even.*

Since the number of colors necessary to distance-k color in this technique is equal to the lower bound on the number of colors established in the previous theorem, this technique produces an *optimal* coloring.

As seen in figure 4 the size of the distance-2 clique is 7 and the size of distance-3 clique is 12. The structure of the distance-k clique of cellular graphs for odd and even values for k is clear from figure 4.

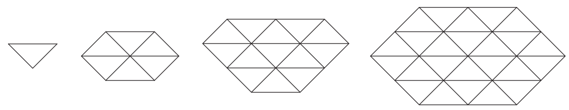

Figure 4. Distance-k cliques for $k = 1, 2, 3, 4$ in a cellular graph.

Step 1: Divide the nodes of a cellular graph $G = (V, E)$, into disjoint node sets of size $3/4(k+1)^2$ if k is odd and $3/4(k+1)^2+1/4$ if k is even. Figures 5 and 6 depict this division of node sets for $k = 2$ and $k = 3$ respectively.

Step 2: Assign colors numbered from 0 through $3/4(k + 1)^2 - 1$ if k is odd and 0 through $3/4(k^2 + 2k)$ if k is even, row after row in each of the disjoint node sets. Figures 5 and 6 depict this assignment of colors for $k = 2$ and $k = 3$ respectively.

Proof of correctness of the color assignment:

We will refer to the disjoint node sets as *blocks*. Two blocks B_i and B_j are said to be adjacent to each other if any node u in the block B_i is adjacent to any node v in the block B_j. A color assignment is *incorrect* only if two nodes u and v are assigned the same color even though the distance between them is less than or equal to k. The nodes u and v are then said to be in *coloring conflict*. It is clear that a coloring conflict between nodes u and v can potentially take place only if u and v are in adjacent blocks. In this coloring scheme an identical coloring assignment is made in every block.

Consider two adjacent blocks B_i and B_j. Because of the structural symmetry, every node in B_i has a corresponding node in B_j. Suppose u is a node in B_i and v is the corresponding node in B_j. Consider a shortest path P from u to v in the cellular graph such that the *longest segment* of the path passes through the nodes of B_i or B_j. Suppose $P = u \rightarrow \cdots \rightarrow x \rightarrow y \rightarrow \cdots \rightarrow v$, where

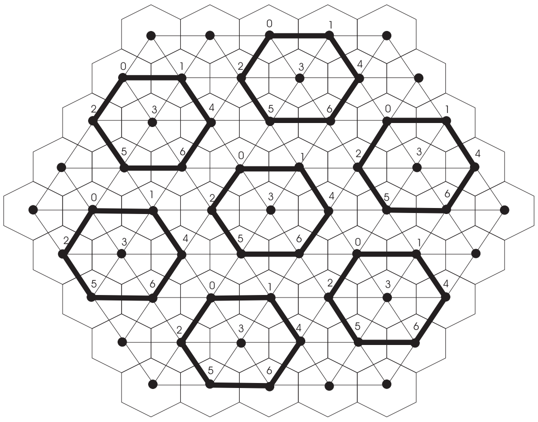

Figure 5. Distance-k coloring of cellular graph with k = 2.

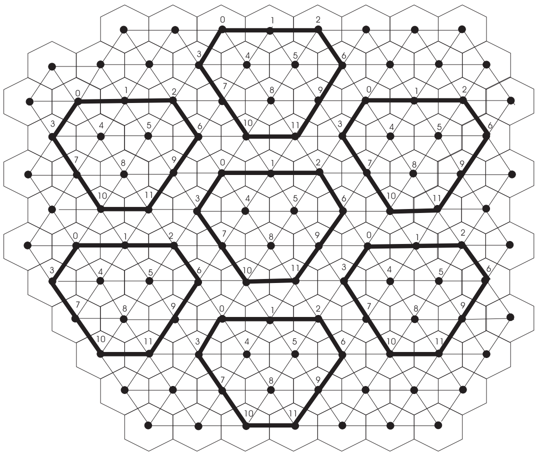

Figure 6. Distance-k coloring of cellular graph with k = 3.

x and y are the nodes on this path where P meets the boundary of the blocks B_i and B_j respectively. Every shortest path P' between u and v will have a boundary node for B_i and one for B_j. Suppose these nodes are referred to as x' and y'. Since P uses the longest part of the path through the nodes of B_i and B_j, this means that $d(u,x)+d(y,v) \geq d(u,x')+d(y',v)$ for all shortest paths P' where $d(a,b)$ is the shortest path length between a and b. Suppose that z is the node in block B_i corresponding to the node y in B_j. Since u and z in B_i corresponds to v and y in B_j, $d(y,v) = d(z,u)$. Therefore, $d(u,x) + d(y,v) = d(u,x) + d(z,u) = d(z,x)$. Since the maximum distance between z and u is k, if in P $d(u,x) + d(y,v) < k$, then we can construct another shortest path P' between u and v such that $d(u,x') + d(y',v) > d(u,x) + d(y,v)$, contradicting the choice of the path P. Therefore, $d(u,x) + d(y,v) = k$ and $d(u,v) = d(u,x) + d(x,y) + d(y,v) = k + 1$ and the same color can be assigned to the nodes u and v without any coloring conflict.

Case II: 2-band buffering system with frequency separation $s_0 \geq s_1 \geq s_2$.

In this section we restrict our attention to a cellular network where interference extends only up to two cells from the call originating cell. In most of the real cellular networks this often is the case. Adjacent channel interference dictates that calls in the cells that are distance one apart should be separated by s_1 and cells that are distance two apart should be separated by s_2. As we have assumed w_i = 1 for all cells, the separation parameter s_0 plays no role in this situation.

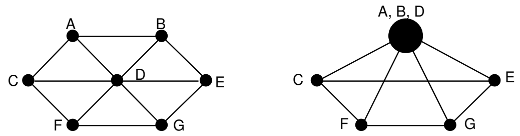

Figure 7. Lower bound of frequency assignment with 2-band interference

Theorem 2 *The lower bound on frequency assignment in a homogeneous network ($w_i = 1$) and 2-band buffering system with frequency separation $s_0 \geq s_1 \geq s_2$ is at least $2s_1 + 4s_2 + 1$.*

Proof: It is easy to see that the seven node graph $(A, \ldots, G)$ shown in figure 7 is a subgraph of a cellular graph. Every node in this subgraph is within distance two of each other. Therefore, they are going to influence each other in their frequency assignment. Within this subgraph, consider the subgraph induced by the nodes A, B and D. These nodes are mutually adjacent. As such frequency assignment on any two nodes in this subgraph must be separated by at least s_1. The assignment on A, B and D can be $0, s_1$ and $2s_1$, requiring a bandwidth of at least $2s_1 + 1$. If we collapse the nodes A, B and D into a *supernode*, we have a five node graph as shown in figure 7. Every node in this graph is within distance two of each other. As such frequency assignment on any two nodes in this subgraph must be separated by at least s_2. This means a lower bound on the frequency assignment on this subgraph is at least $4s_2 + 1$. However, the supernode (A, B, D) cannot be assigned just one frequency. Instead the minimum bandwidth for frequency assignment on A, B and D is $2s_1 + 1$. Therefore, combining these two observations, we can see that the minimum bandwidth needed to assign frequency to the nodes A through G is at least $2s_1 + 4s_2 + 1$.

Frequency Assignment Scheme for a 2-band buffering system with frequency separation $s_0 \geq s_1 \geq s_2$

Considering a seven node subgraph of the cellular network shown in figure 7, the lower bound on frequency assignment was shown to be $2s_1+4s_2+1$ in the previous section. In this section we first show that frequencies can indeed be assigned to the nodes of the seven node subgraph without violating the frequency separation requirements such that the bandwidth is $2s_1 + 4s_2 + 1$. This assignment is shown in figure 8(a), where each node is given a label of the form (p, q). If a node has a label (p, q), it means that the corresponding cell has been assigned the frequency $ps_1 + qs_2$. We refer to the seven node subgraph as a *block*.

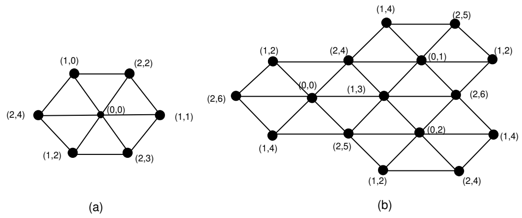

Figure 8. Frequency assignment in blocks

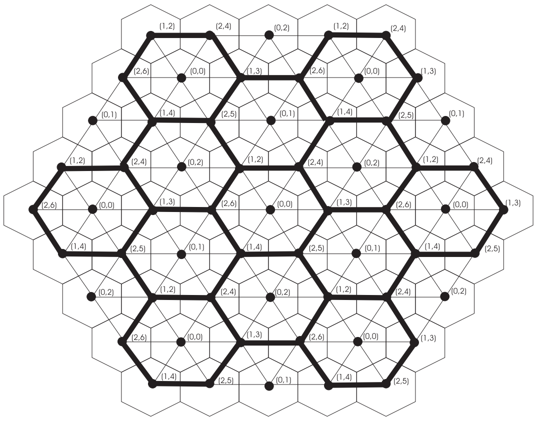

Figure 9. Frequency assignment with 2-band interference

Our idea of assigning frequencies to the nodes of the cellular graph is to repeat the frequency assignment in a block over the entire network. However, this assignment cannot completely be repeated as the center nodes in two adjacent blocks (blocks sharing a common edge), both cannot have a frequency label of (0,0). This is true because the distance between the center nodes of two adjacent blocks is two and as such they must be separated by at least s_2. To deal with this problem we develop another assignment that uses a bandwidth of $2s_1 + 6s_2 + 1$ for a block ($2s_2$ more than the minimum required). However, we can now use this assignment to alleviate the center frequency conflict between adjacent blocks by assigning them labels (0,0), (0,1) and (0,2) as shown in figure 8(b). This structure is a combination of three adjacent blocks and will be referred to as a *superblock*. It is easy to see that the entire cellular network can be decomposed into a set of superblocks and the same frequency assignment for a superblock can be repeated over and over again. Thus the entire network can be assigned frequencies with bandwidth $2s_1 + 6s_2 + 1$. This assignement is shown in figure 9.

Theorem 3 *The bandwidth required by the above scheme may be at most 4/3 times the optimal bandwidth. If $s_1 \geq 2s_2$, then the scheme requires at most 5/4 times the optimal bandwidth.*

Proof: The coloring scheme described above does not require a bandwidth of more than $2s_1 + 6s_2 + 1$. The lower bound for the bandwidth in a 2-band buffering system with homogeneous traffic is $2s_1 + 4s_2 + 1$. Given that $s_1 \geq s_2$, the worst case ratio between these numbers is 4/3, when $s_1 = s_2$. If $s_1 >> s_2$, the ratio is 1 and when $s_1 \geq 2s_2$, the ratio is less than or equal to 5/4.

5 Conclusion

In this paper we have presented an optimal algorithm for a version of the channel assignment problem in a cellular network. In most of the real cellular networks, the interference extends up to two adjacent cells. For this situation we have presented a near optimal algorithm that would require a bandwidth of at most 4/3 times the optimal bandwidth. The frequency separation requirement s_1 between two adjacent cells and $s - 2$ between two cells that are distance two apart is often related by $s_1 \geq 2s_2$. In such a situation, the bandwidth required by this frequency assignment scheme is at most 5/4 times the optimal bandwidth. If s_1 is much larger than s_2 the bandwidth required by this scheme approaches the optimal bandwidth. The results of this paper are directly applicable to *homogeneous* networks. The results are also useful in reducing the problem size for a *non-homogeneous* network as is shown in [20].

References

[1] F. Box, "A Heuristic Technique for Assigning Frequencies to Mobile Radio Nets," *IEEE Transactions on Vehicular Technology*, vol. VT-27,pp. 57-64, May 1978.

[2] M. Duque-Anton, D. Kunz and B. Ruber, "Channel Assignment for Cellular Radio Using Simulated Annealing," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 1, pp.14-21, Feb 1993.

[3] A. Gamst, "Homogeneous Distribution of Frequencies in a Regular Hexagonal Cell System," *IEEE Transactions on Vehicular Technology*, vol. VT-31,pp. 132-144, Aug 1982.

[4] A. Gamst, "Some Lower Bounds for a Class of Frequency Assignment Problems," *IEEE Transactions on Vehicular Technology*, vol. VT-35, no. 1, pp.8-14, Feb 1986.

[5] M.R. Garey and D.S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness. W.H. Freeman Publishers, San Francisco, 1978.

[6] W. K. Hale, "Frequency Assignment: Theory and Applications," *Proceedings of the IEEE*, vol. 68, no. 12, pp.1497-1514, Dec 1980.

[7] S. Jordan and E.J. Schwabe, " Worst-case Performance of Cellular Channel Assignment Policies," *Wireless Networks*, 2 , pp. 265-275, 1996.

[8] S. Khanna and K. Kumaran, "On Wireless Spectrum Estimation and Generalized Graph Coloring," *Proc. of IEEE INFOCOM'98*, April, 1998.

[9] S. Kim and S. Kim, "A Two-Phase Algorithm for Frequency Assignment in Cellular Mobile Systems," *IEEE Transactions on Vehicular Technology*, vol. 43, no. 3, pp.542-548, Aug 1994.

[10] D. Kunz, "Channel Assignment for Cellular Radio using Neural Networks," *IEEE Transactions on Vehicular Technology*, vol. 40, no. 1, pp.188-193, Feb 1991.

[11] R. A. Leese, "A Unified Approach to the Assignment of Radio Channels on a Rectangular Hexagonal Grid," *IEEE Transactions on Vehicular Technology*, vol. 46, no. 4, pp. 968-980, Nov 1997.

[12] V. H. Macdonald, "The Cellular Concept," *The Bell System Technical Journal*, vol. 58, no. 1, Jan 1979.

[13] R. Mathar and J. Mattfeldt, "Channel Assignment in Cellular Radio Networks," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, pp. 647-656, Nov 1993.

[14] L. Narayanan and S. Shende, "Static Frequency Assignment in Cellular Networks," *Proc. of the 4th International Colloquium on Structural Information and Communication Complexity*, 1997.

[15] S. Sarkar and K. N. Sivarajan, "Channel Assignment Algorithms Satisfying Cochannel and Adjacent Channel Reuse Constraints in Cellular Mobile Network," *Proc. of IEEE INFOCOM'98*, April, 1998.

[16] A. Sen, T. Roxborough and S. Medidi, "Upper and Lower Bounds of a Class of Channel Assignment Problems in Cellular Networks," *Proc. of IEEE INFOCOM'98*, April, 1998.

[17] M. Sengoku, H. Tamura, S. Shinoda, T. Abe and Y. Kajitani, "Graph Theoretical Considerations of Channel Offset Systems in a Cellular Mobile System," *IEEE Transactions on Vehicular Technology*, vol. 40, no. 2, pp.405-411, May 1991.

[18] K. N. Sivarajan, R. J. McEliece and J. W. Ketchum, "Channel Assignment in Cellular Radio," *Proc. 39th IEEE Vehicular Technology Conf.*, pp.846-850, May 1989.

[19] D. Tcha, Y. Chung and T. Choi, "A New Lower Bound for the Frequency Assignment Problem,"*IEEE/ACM Transactions on Networking*, vol. 5, no. 1, February 1997.

[20] W. Wang and C. K. Rushforth, "An Adaptive Local-Search Algorithm for the Channel-Assignment Problem," *IEEE Transactions on Vehicular Technology*, vol. 45, no. 3, pp.459-466, Aug 1996.