Survivable Routing in WDM Networks

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Abstract

In this paper we consider the problem of routing the lightpaths of a logical topology of a WDM network on an arbitrary physical topology, such that the logical topology remains connected even after the failure of a physical link. In a recent paper, Modiano et. al. *introduced the notion of* survivable routing *and established a necessary and sufficient condition for the existence survivable routes of a logical topology in a physical topology. In this paper we show that problem of determining whether survivable routing is possible for a logical topology in a given physical topology is an NP-Complete problem. Moreover, we show that the problem remains NP-complete, even when the logical topology is restricted to be a ring with a specific ordering of the nodes.*

1 Introduction

Survivabilty of high bandwidth optical networks has become an important area of research in recent times due to its tremendous importance as a national and international infrastructure for moving large volumes of data from one part of the globe to another. A significant number of papers addressing various aspects of survivability have appeared in the literature [2, 3, 5, 6, 7]. Many of these studies focus on restoration mechanisms to restore all lightpaths in the event of physical link failure. In a recent paper [6], Modiano and Narula-Tam introduced the notion of *survivable routing*, where the objective is to route the lightpaths (corresponding to the links of a logical topology) in the physical topology in such a way, that failure of any single physical (optical fiber) link cannot disconnect the logical network. As defined in [6], a routing is known as *survivable* if the failure of any physical link leaves the (logical) network *connected*. A somewhat similar problem was studied in [2, 7] where the objective was to support IP networks over WDM networks. They studied the problem of layout of IP network topology over the WDM network so that the IP network remains connected under single fiber-link failure.

We illustrate the concept of *survivable routing* with the help of an example given in [6]. Suppose that the logical topology is a ring (figure 1a) and the physical topology is the network shown in figure 1b. The logical topology may be embedded in the physical topology in many different ways. For the embedding, links of the logical topology have to be mapped onto the *paths* in the physical topology. If the logical link (u_1-u_2) is mapped onto the physical path ($v_1 - v_2 - \ldots - v_k$), we denote it by $L(u_1 - u_2) \to P(v_1 - v_2 - \ldots - v_k)$. Consider the following embedding of the logical topology (figure 1a) into the physical topology shown in figure 1b.

Two possible embedding of the logical topology in the physical topology are as follows:
Embedding 1: $L(1-2) \to P(1-2), L(2-3) \to P(2-3), L(3-4) \to P(3-4), L(4-5) \to P(4-5), L(5-1) \to P(5-4-1)$.
Embedding 2: $L(1-2) \to P(1-2), L(2-3) \to P(2-3), L(3-4) \to P(3-4), L(4-5) \to P(4-5), L(5-1) \to P(5-3-1)$.

It can be observed that in embedding 1, failure of the fiber link between the nodes 4 and 5 will disconnect the logical topology, whereas no single link failure will disconnect the logical topology in embedding 2. Thus it is clear that the way the lightpaths are routed has a tremendous impact on the survivability of the logical network. In the example, survivable routes could be found if the logical links were mapped on the physical paths in the "correct" way and could not be found if they were not. However, for certain physical topologies there may not be any "correct" link mapping and as such survivable routes for a logical topology cannot be found in such physical topologies. Survivable routes for the logical ring in figure 1a cannot be found if the physical topology is as shown in figure 1c. In this paper we examine the issues related to the existence of a survivable routing of a logical topology in a physical network of arbitrary topology.

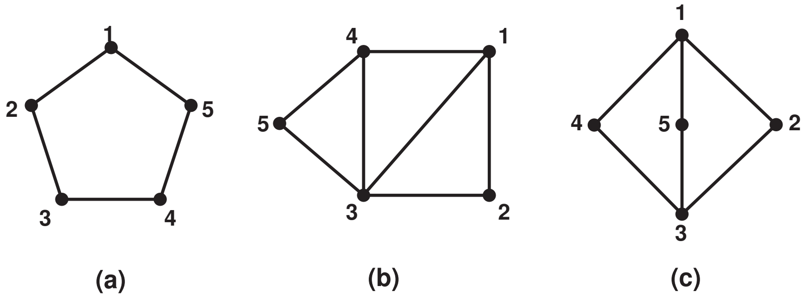

Figure 1. Logical and physical topologies of WDM networks

It may be noted that as in [6], we assume in this paper that both the logical and physical topologies of the network are given and we try to find if survivable routes for the given logical topology exist in the given physical topology. A number of researchers have examined the issues of protection and restoration in WDM network and have suggested the use of *protection cycles* [3] and other techniques [5]. However, as noted in [6], in many cases providing protection at the optical as well as electronic layer may be somewhat redundant and a less stringent requirement that the network remain connected in spite of the failure of any single fiber link may be sufficient. This paper addresses the issues related to such cases.

2 Problem Formulation

The physical topology of the network is represented by an *undirected* graph $G_p = (V_p, E_p)$, where V_p is the set of nodes and E_p is the set of *physical* links. Similarly, the logical topology is represented by another undirected graph $G_l = (V_l, E_l)$, where V_l is the set of nodes and E_l is the set of *logical* links. We assume that $V_p = V_l$. The objective of the survivable routing problem is to find a way to route (map) the logical topology on the physical topology such that the logical topology remains connected inspite of the failure of any one single physical link.

In order to establish a logical link between the nodes s and t of the logical network, a lightpath needs to be established between the nodes $f(s)$ and $f(t)$ in the physical network, where $f(s)$ and $f(t)$, are the images (or mappings) of the nodes s and t in the physical network. Such a lightpath may use a set of physical links and some wavelengths on these links. Since the objective of this paper is to focus on the issues of a survivable design, as in [6], we assume that either a sufficient number of wavelengths or a sufficient number of wavelength converters are available, so that the issues related to wavelength continuity can be ignored. It may be noted that it is possible that $f(s) = s$ and $f(t) = t$.

Survivable Routing Problem (SRP)
Instance: Two connected undirected graphs $G_p = (V_p, E_p)$ and $G_l = (V_l, E_l)$ representing the physical and the logical topology of the network respectively, ($V_p = V_l$).
Question: Is it possible to map the edges of G_l into paths of G_p such that removal of any one link from G_p would not disconnect G_l?

We will prove that SRP is NP-complete, by considering a restricted version of the problem when the logical topology is a ring. We will consider two different versions, (i) node i of the logical topology is mapped to node i of the physical topology for all i, and (ii) node i of the logical topology is not necessarily mapped to node i of the physical topology. In our notation the first situation is described by $f(s) = s$.

We use standard graph theoretical terminologies from [1]. For the convenience of our proof, we consider closed *trails* [1] with different origins to be different. For example, $1 \to 2 \to 3 \to 1 \to 4 \to 5 \to 1$ is considered to be a different closed trail from $2 \to 3 \to 1 \to 4 \to 5 \to 1 \to 2$. A graph is called *cubic* if all the nodes in the graph are of degree 3.

3 Survivable Routing of a Logical Ring

Let (f, F) denote a mapping between two graphs, where f is the node mapping, F is the edge-to-path mapping.

Theorem 1 *Let $G_p = (V_p, E_p)$ and $G_l = (V_l, E_l)$ represent the physical and the logical topology of the network respectively. If G_l is a ring network, then there exists a survivable routing for G_l in G_p if and only if G_p contains a closed trail visiting each node at least once.*

Proof: Suppose that the logical ring network is the cycle with n nodes represented as $v_0 e_1 v_1 e_2 v_2 \ldots e_{n-1} v_{n-1} e_n v_0$ such that for $1 \leq i \leq n-1$, the ends of e_i are v_{i-1} and v_i and the ends of e_n are v_n and v_0. Let (f, F) denote a mapping of G_l in G_p, where $f(v_i)$ is the mapping of the nodes of G_l onto the nodes of G_p and $F(e_i)$ is the mapping of the edges of G_l onto the *paths* of G_p (paths may be of length one).

$\Longrightarrow$ Suppose that there is a survivable routing for G_l in G_p given by (f, F). Then for all $e_i \neq e_j$, $F(e_i) \cap F(e_j) = \emptyset$. This is true because, if there was an edge e of the physical graph in $F(e_i) \cap F(e_j)$ (i.e., $e \in F(e_i) \cap F(e_j), e_i \neq e_j$, then failure of e would disconnect the logical links e_i and e_j. Since G_l is a ring network, failure of the links e_i and e_j would disconnect G_l, contradicting the assumption that (f, F) is a survivable routing for G_l in G_p. Thus all $F(e_i)$'s are pair-wise edge-disjoint. It is not difficult to check that $F(e_1)F(e_2)\dots F(e_n)$ forms a closed trail visiting each node of G_l at least once.

$\Longleftarrow$ Suppose that G_p has a closed trail, $w = u(0)u(2)...u(m-1)$, visiting each node at least once. Here, each $u(i)$ is a node of G_p. It may be noted that for some i and j, $u(i)$ may be equal to $u(j)$ even though $i \neq j$. Suppose that $P(u(i), u(j))$ denotes the path between the nodes $u(i)$ and $u(j)$ on w.

Now we construct a mapping (f, F).

```
begin
   for (i := 0; i < m; i := i + 1)
       mark(u(i)) := 0;
   f(v_0) := u(0)
   mark(u(0)) := 1;
   index := 1;
   j := 1;
   while (j < n) do
          {n is the number of nodes in G_l}
       begin
          while (mark(u(index)) = 1) do
              index := index + 1;
          f(v_j) := u(index)
          mark(u(v_j)) := 1;
          index := index + 1;
          j := j + 1 ;
       end
   for (i := 1; i < n; i := i + 1)
       F(e_i) := P(f(v_{i-1}), f(v_i));
       /*P(f(v_{i-1}), f(v_i)) denotes the
       path from the node f(v_{i-1}) to the node
       f(v_i) on the closed trail,
       w = u(0)u(2)...u(m - 1)
       in the physical network G_p */
   F(e_n) := P(f(v_{n-1}), f(v_0));
end
```

We claim that (f, F) is a survivable routing. First, since w visits each node of G_p at least once, $f()$ is a mapping from V_l to V_p. Since w is a trail, the way the function $F()$ is constructed, $F(e_i) \cap F(e_j) = \emptyset$ for all i and j when $i \neq j$. Therefore, the failure of one link in G_p will disconnect at most one edge in G_l, leaving the logical network (ring) connected. Thus (f, F) is survivable.

As noted earlier an *Euler tour* is a tour which traverses each edge *exactly once*. Since such a tour traverses each edge exactly once, it must be traversing each node at least once. The necessary and sufficient condition for a graph to have an Euler tour is given by the following theorem [1].

Theorem 2 *A nonempty connected graph is eulerian if and only if it has no nodes of odd degree.*

As stated in theorem 1, it is possible to find a survivable routing for a logical ring network in a physical network of arbitrary topology, if and only if the physical network $G_p = (V_p, E_p)$ contains a closed trail visiting each node at least once. Suppose that $G'_p = (V'_p, E'_p)$ is a subgraph of G_p such that (i) $V_p = V'_p$ and $E'_p \subseteq E_p$ and G'_p is eulerian.

It is not difficult to verify that $G_p = (V_p, E_p)$ contains a closed trail visiting each node at least once, if and only if such a subgraph G'_p of G_p exists. From theorem 2 it is known that G'_p will be eulerian if and only if G'_p is connected and has no node with an odd degree. It may be noted that the physical network G_p may or may not have such a subgraph G'_p. Thus a survivable routing for a logical ring network in a physical network, G_p, of arbitrary topology exists if and only if G_p contains such a subgraph G'_p.

Survivable Routing of Ring Problem (SRRP)
Instance: A connected undirected graph $G = (V, E)$.
Question: Does G have a subgraph $G' = (V', E')$, such that $V = V'$ and $E' \subseteq E$, G' is connected and has no node with an odd degree.

We prove that SRRP is NP-complete by restricting it to cubic graphs (it may be recalled that a graph is called *cubic* if all the nodes in the graph are of degree 3)

Survivable Routing of Ring Problem in Cubic Graph (SRRPG)
Instance: A connected undirected cubic graph $G = (V, E)$.
Question: Does G have a subgraph $G' = (V', E')$, such that $V = V'$ and $E' \subseteq E$, such that G' is connected and has no node with an odd degree ?

Hamiltonian Cycle Problem (HCP)
Instance: A connected undirected graph $G = (V, E)$.
Question: Does G contain a Hamiltonian Cycle ?

Hamiltonian Cycle Problem in Planar, Cubic and Triply connected Graph (HCPPCT)
Instance: A connected undirected graph $G = (V, E)$, which is (i) planar, (ii) cubic and (iii) triply connected (i.e., deletion of any two nodes leaves the graph connected).
Question: Does G contain a Hamiltonian Cycle ?

It has been proven in [4] that HCPPCT is NP-complete.

Hamiltonian Cycle Problem in Cubic Graph (HCPC)
Instance: A connected undirected cubic graph $G = (V, E)$.

Question: Does G contain a Hamiltonian Cycle ?

HCPC is NP-complete because HCPPCT, a restricted version of HCPC is NP-complete.

Theorem 3 *Survivable Routing of Ring Problem in Cubic Graph (SRRPCG) is NP-complete.*

Proof: Clearly, SRRPCG is in NP as it is fairly simple to check if G' is connected and has no nodes with odd degree.

We will give a transformation from the known NP-complete problem HCPC. We take the instance of SRRPCG to be the same as the instance of HCPC. Suppose that this instance is $G = (V, E)$. We claim that G has a Hamiltonian cycle if and only if there is subgraph $G' = (V', E')$ in G that is connected and has no node with an odd degree.

Suppose that $G = (V, E)$ contains a Hamiltonian cycle. Construct a subgraph $G' = (V', E')$ as follows: $V' = V$ and E' is the set of edges that make up the Hamiltonian cycle. Clearly, G' is connected and has no node with an odd degree (all nodes of G' have degree 2).

Conversely, suppose that $G = (V, E)$ has a subgraph $G' = (V', E')$ such that $V' = V$, $E' \subseteq E$ and G' has no node of odd degree. Since G is a cubic graph all the nodes of G are of degree 3. Since G' does not have any node with an odd degree, all nodes of G' must be of degree 2. Since all nodes of G' are of even degree (2), G' has a Euler tour. Since all nodes of G' are of degree 2, this Euler tour is also a Hamiltonian cycle of G' and hence G. This proves the theorem.

Theorem 4 *Survivable Routing of Ring Problem is NP-complete.*

Proof: SRRP is NP-complete because SRRPCG, a restricted version of SRRP is NP-complete.

In the next section, we consider the situation $f(s) = s$, i.e., the nodes of the logical topology is labeled exactly the same way as the nodes of the physical topology.

4 Ordered Traversal of Closed Trail (OTCT) Problem

In this section, we consider an "ordered" version of the survivable routing of rings. In the previous discussion of this paper, the node-mapping, from V_l onto V_p is not given. Now, suppose G_l is still a ring, and the node-mapping, f, is given, we try to see if there exists an edge-mapping, F, from E_l to E_p, such that (f, F) forms a survivable routing for G_l in G_p. Suppose $|V_l| = |V_p| = n$, and nodes in V_l are labeled as $1, 2, ..., n$, where i is adjacent to $i + 1$, for all $i \in \{1, ..., n-1\}$, and n is adjacent to 1. Without loss of generality, we can also let $V_p = \{1, 2, ...n\}$. Then, a node-mapping is nothing but a permutation (an order), π, of V_p (or say the set $\{1, 2, ..., n\}$); and to find an edge-mapping is the same as to find a set of paths, P_i, such that P_i connects $\pi(i)$ and $\pi(i+1)$, for all $i \in \{1, ..., n-1\}$, and P_n connects $\pi(n)$ and $\pi(1)$.

By the proof of Theorem 4, the P_i's must be pairwise edge-disjoint. Hence, $\pi(1)P_1\pi(2)P_2...\pi(n-1)P_{n-1}\pi(n)P_n\pi(1)$ is a closed trail, which contains $\pi = \pi(1)\pi(2)...\pi(n)$ as a subsequence.

Therefore, we have the following corollary of Theorem 4.

Corollary 5 *: Let $G_p = (V_p, E_p)$ and $G_l = (V_l, E_l)$ represent the physical and the logical topology of the network respectively. Suppose that the node mapping is such that $f(v) = v$, for all nodes. Suppose that G_l is a ring network and π is the permutation of the nodes of V_l of that defines the interconnection of G_l. In this case, there exists an edge-mapping, F, from E_l to E_p such that (f, F) forms a survivable routing for G_l in G_p if and only if G_p contains a closed trail, which contains π as a subsequence.*

Ordered Traversal of Closed Trail Problem
Instance: A connected undirected graph $G = (V, E)$, a permutation π of the nodes of V.
Question: Is there a closed trail CT in G, such that the nodes of G appear in the order specified by π, i.e. π is a subsequence of CT ? (Note: the appearance in a sequence does not necessarily imply the first appearance.)

We prove OTCT is NP-complete by giving a transformation from Edge Disjoint Paths Problem .

Edge Disjoint Paths Problem (EDP)
Instance: An undirected graph $G = (V, E)$, collection of disjoint vertex pairs, $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$.
Question: Does G contain k mutually edge-disjoint paths, one connecting s_i and t_i for each i, $1 \le i \le k$?

It is known that EDP is NP-complete.

Theorem 6 *Ordered Traversal of Closed Trail Problem (OTCT) is NP-complete.*

Proof: Clearly, the problem is in NP, as it is fairly simple to check if the nodes of G appear in the closed trail in the order specified by π, i.e., if π is a subsequence of the closed trail.

We give the transformation from the Edge Disjoint Paths problem which is known to be NP-complete.

Given a graph $G = (V, E)$, suppose S and T are two disjoint subsets of the vertex set V and $S = \{s_1, s_2, \ldots, s_k\}$ and $T = \{t_1, t_2, \ldots, t_k\}$, $S \cap T = \emptyset$. We create an instance of the OTCT problem from an instance of the EDP problem. From the graph $G = (V, E)$, which is part of the instance of the EDP problem, we create the graph $G^+ = \{V \cup V', E \cup E'\}$, which is a part of the instance of

the OTCT problem in the following way:
$\forall i \in \{1, \ldots, k-1\}$, we introduce a new node $u_i \in V'$ and two new edges $(t_i, u_i) \in E'$ and $(u_i, s_{i+1}) \in E'$. The new node and edges establish a path from the node t_i to the node s_{i+1} through the node u_i. We refer to this path as Q_i. Suppose that $A = V - S - T$, i.e., the set of nodes of G that are neither in S nor in T.
Case A: $A = \emptyset$. Introduce a new node $u_k \in V'$ and connect t_k and s_1 by a path $Q_k : t_k - u_k - s_1$.
Case B: $A \neq \emptyset$. Suppose that $A = \{v_1, v_2, \ldots, v_a\}$. Introduce new nodes $w_0, w_1, \ldots, w_{a-1}, w_a$ and a new path $Q_k : t_k - w_0 - v_1 - w_1 - v_2 \ldots - w_{a-1} - v_a - w_a - s_1$.

The construction of G^+ is now complete. Now we give a permutation π of the node set $V \cup V'$. If $A = \emptyset$, $\pi = s_1 t_1 u_1 s_2 t_2 u_2, \ldots, t_{k-1} u_{k-1} s_k t_k u_k$,
If $A \neq \emptyset$, $\pi = s_1 t_1 u_1 s_2 t_2 u_2, \ldots, t_{k-1} u_{k-1} s_k t_k w_0 v_1 w_1 v_2 w_2, \ldots, w_{a-1} v_a w_a$.

Clearly, the construction of G^+ and the permutation π can be carried out in polynomial time.
Claim: $\exists$ a closed trail CT in G^+ such that π is a subsequence of CT, if and only if $\exists$ a set of edge-disjoint paths in G connecting the nodes (s_i, t_i) for $1 \leq i \leq k$.
$\Leftarrow$ Suppose that there exists paths $P_1, \ldots, P_k$ connecting the node pairs (s_i, t_i) for $1 \leq i \leq k$ respectively. Then clearly, $P_1 Q_1 P_2 \cdots Q_{k-1} P_k Q_k$ is a closed trail in G^+ preserving the order of π.
$\Rightarrow$ Suppose $\exists$ such a closed trail CT in G^+. $\forall i \in \{1, \ldots, k\}$, let W_i be the walk along CT, connecting the node s_i with the node t_i. We obtain P_i from W_i by removing all cycles contained in W_i. Obviously, P_i connects s_i with t_i $\forall i, 1 \leq i \leq k$. Furthermore, $P_i's$ are edge disjoint. Now, the only thing we need to show is $P_i \subseteq G$ $\forall i, 1 \leq i \leq k$, i.e., the path $P_i, 1 \leq i \leq k$, do not use the nodes $u_i, 1 \leq i \leq k$, and $w_i, 1 \leq i \leq a$. Suppose $\exists i$ such that $P_i \not\subseteq G$, which implies that P_i visits some of the new nodes of the form u_i or w_i. But the degree of these nodes is two, which means that these nodes can be visited exactly once in CT. Recall that the permutation is either $\pi = s_1 t_1 u_1 s_2 t_2 u_2, \ldots, t_{k-1} u_{k-1} s_k t_k u_k$, or $\pi = s_1 t_1 u_1 s_2 t_2 u_2, \ldots, t_{k-1} u_{k-1} s_k t_k w_0 v_1 w_1 v_2 w_2, \ldots, w_{a-1} v_a w_a$. According to the permutation π, the u_i and w_i nodes have to be visited once outside W_i in CT. If these nodes are traversed once within W_i, it implies that these nodes were traversed twice in CT, contradicting our assumption of CT being a trail. Therefore, $\{P_1, \ldots, P_k\}$ is a set of edge-disjoint paths in G, connecting s_i with t_i.

The above theorem shows that it is NP-complete to determine if survivable routes exists for a logical topology G_l, in an arbitrary physical topology G_p, even when G_l is a ring. Since this restricted problem (G_l restricted to be a ring) is NP-complete, the more general Survivable Routing Problem (section 2), where topology of G_l can also be arbitrary, is NP-Complete.

5 Mathematical Programming Formulation

A mathematical programming [8] solution to the optimization version of the survivable routing problem is given next. This formulation seeks to find a set of k mutually edge disjoint paths between a set of k source-destination node pairs $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$.

The integer linear programming formulation for establishing edge disjoint paths between k source-destination node pairs $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$ is given below.

The variable r is used to identify a path, $1 \leq r \leq k$. A binary indicator variable $x^r_{i,j}$ is associated with each link (i, j) of the graph $G = (V, E)$. If the variable $x^r_{i,j} = 1$, it indicates that the link (i, j) is a part of the r-th path from the source to the destination and if $x^r_{i,j} = 0$ then it is not a part of such a path.

$$Minimize \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{r=1}^{k} x^r_{i,j}$$

Subject to the following constraints:

$$(i) \forall j \in V, \sum_{\{i|i \in V\}} x^r_{i,j} - \sum_{\{i|i \in V\}} x^r_{j,i} = \begin{cases} -1 & \text{if } j \text{ is the source} \\ 1 & \text{if } j \text{ is the destination} \\ 0 & \text{if } j \text{ is any other node} \end{cases}$$

$$(ii) \sum_{r=1}^{k} x^r_{i,j} \ \leq \ 1, \ \forall \ (i,j) \in E$$

$$(iii) \ x^r_{i,j} \ = \ 0/1, \ \forall \ r, \ 1 \leq r \leq k; \ \forall (i,j) \in E$$

If the above ILP produces a solution, it implies that survivable routes exist for the logical topology in the given physical topology. However, if the ILP fails to produce a solution, we in general, cannot infer that such routes do not exist. However, if logical topology is a ring, we can draw such an inference. The main problem with the above formulation is that the variables $x^r_{i,j}$ are required to take only 1/0 values, indicating whether the link is a part of the path or not. A fractional value $x^r_{i,j}$, (e.g. $x^r_{i,j} = 0.37$) does not have a physical meaning. The computational complexity of the ILP problems is exponential, whereas LP problems can be solved in polynomial time by algorithms using interior point methods [8]. If the constraint (iii) could be relaxed in the following way,

$$(iv) \ x^r_{i,j} \ \geq \ 0, \ \forall \ r, \ 1 \leq r \leq k; \ \forall (i,j) \in E$$

the ILP problem reduces to an LP problem, which can be solved in polynomial time. However, relaxing the constraint (iii) and replacing it with (iv), leaves open the possibility of finding a solution where $x^r_{i,j}$ has a fractional value (e.g.

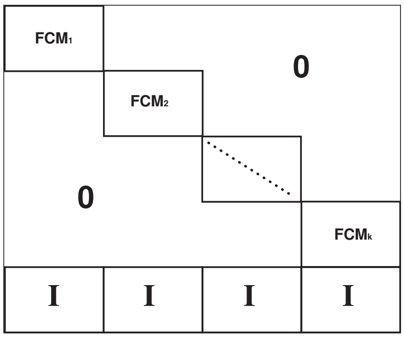

Figure 2. Structure of the constraint matrix of the disjoint path problem; FCM_i: Flow conservation matrix for (s_i, t_i); I: Identity matrix.

$x^r_{i,j} = 0.37$). However, such a solution will never be obtained in case the constraint matrix of the LP formulation has a specific structure.

Definition: Totally Unimodular Matrix: A matrix A is *totally unimodular* if each square submatrix of A has determinant 0, +1 or -1 [8].

It is well known that if the constraint matrix of a LP problem is totally unimodular, then the polytope defined by the constraint matrix has integral extreme points only [8], and as such integral solutions to the problem can be obtained without having explicit integrality constraints, such as constraint (iii) in the above formulation.

The set of constraints given in (i) in the above formulation for each $r, 1 \leq r \leq k$, is known as the *flow conservation* constraints. These constraints essentially set up a path from s_r to t_r for each $r, 1 \leq r \leq k$. We refer to the constraint matrix formed by the constraints for a specific r as the flow control matrix $FCM_r, 1 \leq r \leq k$. It may be observed that FCM_r's are nothing but a *node-arc incidence* [8] matrix of the physical topology of the WDM network. The structure of the constraint matrix with constraints (i) and (ii) is as shown in figure 2, where $FCM_r, 1 \leq r \leq k$, represent the flow conservation matrices associated with each path from s_r to t_r for $1 \leq r \leq k$ and I represents an identity matrix. As the FCM_r's are node-arc incidence matrix, from the following theorem, we conclude that FCM_r's are totally unimodular matrices.

From [8] (pp. 280) it is known that total unimodularity is preserved under the *1-sum* composition, defined as follows:

$$A \oplus_1 B = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$

i.e., if A and B are totally unimodular matrix then so is the matrix obtained by performing 1-sum operation. By examining the structure of the constraint matrix shown in 2, it is clear that matrix formed by $FCM_i, 1 \leq i \leq k$ form a totally unimodular matrix (as each $FCM_i, 1 \leq i \leq k$, is individually totally unimodular). Because of the disjointness constraint among the paths, the lower part of the constraint matrix of figure 2 comprises of k identity matrices. Because of the presence of these identity matrices, the complete matrix of figure 2 is no longer guaranteed to be totally unimodular. However, because for the most part the matrix of figure 2 maintains totally unimodular property, we *conjecture* that *most* of the extreme points of the polytope defined by this matrix are *integral* points. This implies that if our conjecture is correct, we should obtain integral solutions for the disjoint path problem, most of the time, even when we execute the problem by relaxing the integrality constraint, (i.e. $x^r_{i,j} \geq 0$ instead of $x^r_{i,j} = 0/1$, $\forall\, r,\ 1 \leq r \leq k$; $\forall (i,j) \in E$).

We conducted a series of experiments to test our conjecture. We generated a series of random graphs, with a varying number of nodes, average degree of nodes and number of edge disjoint paths. Using the mathematical programming formulation given earlier, we executed it on a SUN Ultra workstation using CPLEX 6.5 mathematical programming package. We executed each data set, once with the constraint $x^r_{i,j} = 0/1$ and once with $x^r_{i,j} \geq 0$. The results of these experiments showed that almost always optimal solutions were obtained with relaxed condition.

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