Wealth Effects and Agency Costs: Supplementary Material

Hector Chade* Virginia N. Vera de Serio†

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*Arizona State University, Department of Economics, Tempe, AZ 85287-3806.
†Universidad Nacional de Cuyo, Facultad de Ciencias Económicas, I.C.B., Mendoza, Argentina 5500.

1 Introduction

In these notes we provide detailed proofs of results omitted from the text. We start with a comment on the constraint qualification in the cost minimization problem, and then move on to a long section on differentiability of the cost function and the optimal contract. Then we construct an n-outcome example of a principal-agent problem that illustrates the tightness of TW's condition. Next we justify an assertion about the variance of the contract made in the text, as well as one about the small disutility of effort case. Finally, we prove two claims about alternative assumptions on the agent's outside option, and provide proofs of a couple of assertions made about large wealth and a continuum of actions. The Appendix contains a proof of the continuity properties of the cost function and the optimal contract—which requires only first-order continuous differentiability of the primitives—that is used to apply an envelope theorem, and that can be of independent interest.

2 A Remark on the Constraint Qualification

Notice that the cost minimization problem amounts to minimizing a strictly convex function subject to linear constraints. Thus, the 'refined Slater condition' (see Boyd and Vandenbergue (2006) pp. 226-7) reduces to feasibility plus open domain of the objective function in this case (which holds in this problem). Therefore, when the feasible set is nonempty, then the unique optimum is characterized by the Kuhn-Tucker conditions. In short, either the constraint set is empty and there is nothing to solve (the action is not implementable or, equivalently, its cost is infinite), or it is nonempty and no additional regularity condition is needed.

In the continuum of actions case there is a simple sufficient condition to ensure that the feasible set is nonempty: for each a there is an outcome q_i such that $\pi_i'(a) \neq 0$ (this is weaker than assuming the strict MLRP). Similarly, in the finite case with the MLRP and CDFC conditions — in which case only the local downward incentive constraint binds (see Grossman and Hart (1983) p. 34) — it suffices to assume that for each a_k, there is an outcome q_i such that $\pi_i(a_k) \neq \pi_i(a_{k-1})$. To avoid dealing with the uninteresting case in which the constraint set is empty, we make these mild assumptions in these cases.

3 Differentiability Properties of C and v_i

3.1 Continuous Differentiability of C with respect to θ

Both in Thiele and Wambach (1999) and in our paper, the derivative of the cost function with respect to θ plays a fundamental role. We now justfiy it from primitives requiring only first-order continuous differentiability of $V(\cdot)$, $\pi(\cdot)$, and $\psi(\cdot)$.

Consider the following transformation of variables: $z_i = v_i - V(\overline{I} + \theta)$, $i = 1, ..., n$. The problem then becomes:

$$C(a, \theta) = \min_{z_1, \ldots, z_n} \sum_i \pi_i(a) h(z_i + V(\overline{I} + \theta)) - \theta$$

subject to $\sum_i \pi_i(a) z_i - \psi(a) \geq 0$, and $a \in \operatorname{argmax}_{a'} \sum_i \pi_i(a') z_i - \psi(a')$ (this is a finite number of inequalities when A is finite, or a first order condition when $A = [0, \bar{a}]$). Notice that the parameter θ appears now *only* in the objective function.

Grossman and Hart (1983) plus strict convexity of h imply that, for each θ, there is a unique solution, $Z^*(\theta) = \{z^*(\theta)\}$, to the above problem. We show in the Appendix below that $z^*(\cdot)$ is *continuous* in θ.

To see that the cost function is continuously differentiable in θ, restrict the domain of θ to some compact set $[0, \bar{\theta}]$, $\bar{\theta} < \infty$, and consider the set $Z^*([0, \overline{\theta}])$.

Clearly, both the value function and solution to the cost minimization problem are the *same* if we replace the feasible set by $Z^*([0, \overline{\theta}])$, that is, if we solve

$$C(a, \theta) = \min_{(z_1, \ldots, z_n) \in Z^*([0, \overline{\theta}])} \sum_i \pi_i(a) h(z_i + V(\overline{I} + \theta)) - \theta.$$

Moreover, since $z^*(\cdot)$ is continuous in θ, it follows that $Z^*([0, \overline{\theta}]) \subset \mathbb{R}^n$ is *compact*.

Therefore, it is now easy to verify that *all* the assumptions of Corollary 4 part (*iii*) in Milgrom and Segal (2002) are satisfied. Hence, $C(a, \cdot)$ is continously differentiable in

θ, and for any interior θ the derivative is given by

$$\begin{aligned}\frac{\partial C(a,\theta)}{\partial \theta} &= \sum_i \pi_i(a) h'(z^*(a,\theta) + V(\overline{I}+\theta))V'(\overline{I}+\theta) - 1 \\ &= \sum_i \pi_i(a) \frac{1}{V'(I_i+\theta)} V'(\overline{I}+\theta) - 1.\end{aligned}$$

3.2 Continuous Differentiability of C and v_i in (a, η, θ)

In the proof of Proposition 3 we assume that v_i is continuously differentiable in η. We now show, using a straightforward adaptation of Lemma 2 of Jewitt, Kadan, and Swinkels (2008), that if $V(\cdot)$, $\pi(\cdot)$ and $\psi(\cdot)$ are $\mathcal{C}^2$, then both C and v_i are continuously differentiable in (a, θ, η) under MLRP and CDFC. These two properties ensure that, when $A = [0, \bar{a}]$, the first-order approach is valid, and when A is finite, only the local downward incentive constraint is binding when implementing any given action (Grossman and Hart (1983) p. 34). In both cases the constraint set can, without loss of generality, be reduced to two equality constraints.

We will just prove the result for $A = [0, \bar{a}]$, as the finite case is analogous. Consider the cost minimization problem

$$(P) \qquad C(a,\theta,\eta) = \min \sum_i \pi_i(a)\, h(v_i) - \theta$$

$$\begin{aligned} s.t. \qquad \sum_i \pi_i(a)\, v_i &= \overline{v} + \eta \psi(a), \\ \sum_i \pi_i'(a)\, v_i &= \eta \psi'(a). \end{aligned}$$

As mentioned, problem (P) has a unique solution $v_i = v_i(a, \theta, \eta)$, which is characterized by the following system of equations:

$$\begin{aligned} -\pi_i(a)\, h'(v_i) + \pi_i(a)\, \lambda(a,\theta,\eta) + \pi_i'(a)\, \mu(a,\theta,\eta) &= 0, \qquad i = 1, ..., n, \\ \sum_i \pi_i(a)\, v_i &= \overline{v} + \eta\psi(a), \\ \sum_i \pi_i'(a)\, v_i &= \eta \psi'(a), \end{aligned} \tag{1}$$

where $\lambda(a,\theta,\eta)$ and $\mu(a,\theta,\eta)$ are Lagrange multipliers. Therefore

$$v_i = V\left((V')^{-1}\left(\frac{1}{\lambda(a,\theta,\eta)+\frac{\pi_i'(a)}{\pi_i(a)}\mu(a,\theta,\eta)}\right)\right),$$

and $\lambda(a,\theta,\eta)$ and $\mu(a,\theta,\eta)$ are implicitly defined by the last two equations in the system (1). As in Lemma 2 in Jewitt, Kadan, and Swinkels (2008), we can apply the Implicit Function Theorem to conclude that $\lambda(a,\theta,\eta)$ and $\mu(a,\theta,\eta)$ are continuously differentiable for any $a \in A$, $\theta > 0$ and $\eta \geq 0$. Therefore the optimal contract $v_i = v_i(a,\theta,\eta)$ is $\mathcal{C}^1$; obviously, this implies that the same property holds for $C(a,\theta,\eta)$.[1]

3.3 Twice Continuous Differentiability of C and v_i in (a,η,θ)

In Section 4 of this appendix, we assume the existence of the second derivative of the optimal contract with respect to a. To justify this property, we provide here a simple set of conditions that will make the contract twice continuously differentiable. This simple argument we use might be of some independent interest.

We claim that MLRP, CDFC, plus $V(\cdot)$, $\pi(\cdot)$, and $\psi(\cdot)$ three times continuously differentiable imply that the optimal contract is twice continuously differentiable. To justify this assertion, notice that following the same steps as in the previous section, we can apply the Implicit Function Theorem for $\mathcal{C}^k$ functions with $k = 2$ (see Fiacco (1983), Theorem 2.4.1, or Dontchev and Rockafellar (2009), Proposition 1B.5) given that the primitives are three times continuously differentiable. Then the corresponding Lagrange multipliers and the optimal contract $v_i = v_i(a,\theta,\eta)$ are twice continuously differentiable. Moreover, $C(a,\theta,\eta)$ is twice continuously differentiability as well.

4 Tightness of TW's Condition: n–Outcome Case

In the paper, we provide an intuitive proof of Proposition 2 (based on a suggestion by one of the referees), which shows that TW's condition is tight. That proof exploits in a crucial way the assumption of two outcomes, since this allows for a closed form solution

[1] Moreover, notice that the cross partial $\partial C/\partial\theta\partial\eta$ exists in this case, which we use in the proof of Proposition 3. To see this, apply Corollary 4 in Milgrom and Segal (2002) as above to obtain $\partial C/\partial\theta$, and then differentiate with respect to η; only $\partial v_i/\partial\eta$ appears in the resulting expression.

of the optimal contract that implements any action.

In this section we assume n outcomes, and prove that if the principal's cost of implementing an action higher than the lowest one is increasing in the agent's wealth θ for all choices of $(\psi(\cdot), \pi(\cdot), A, Q, \bar{I}, \theta)$, then $V(\cdot)$ satisfies TW's condition. We do so by generalizing the argument that we used in our original proof of the Proposition 2 in the paper (which was done for two possible outcomes only).

Suppose that $V(\cdot)$ is such that $P(\hat{I}+\hat{\theta}) > 3R(\hat{I}+\hat{\theta})$ for some $\hat{I}$ and $\hat{\theta}$. We will show that there exists a principal-agent problem with $V(\cdot)$ as the agent's utility function such that, for some action a, $\partial C(a,\theta)/\partial\theta < 0$ in an open neighborhood of $\hat{\theta}$.

To this end, assume $A = [0, \bar{a}]$; $q_1, ..., q_n$, with $q_i < q_{i+1}$; $\pi(\cdot)$ three times continuously differentiable with $\pi_i(a) > 0$ and $\pi_i'(a)$ not all null for all a; $\psi(\cdot)$ three times continuously differentiable, with $\psi''(0) > 0$; $\bar{I} = \hat{I}$ and $\theta = \hat{\theta}$.

The optimal contract that implements a is characterized by the following system:

$$\begin{aligned} -\pi_i(a)h'(v_i) + \lambda(a)\pi_i(a) + \mu(a)\pi_i'(a) &= 0, \quad i = 1, ..., n, \\ \sum_{i=1}^{n} \pi_i(a)v_i &= \overline{v} + \psi(a), \\ \sum_{i=1}^{n} \pi_i'(a)v_i &= \psi'(a), \end{aligned} \tag{2}$$

where $\lambda(a)$ and $\mu(a)$ are Lagrange multipliers. Now, reasoning as before in Subsections 3.2 and 3.3 we may apply the Implicit Function Theorem for $\mathcal{C}^k$ functions with $k = 2$, to get that $v_i(a)$ is twice continuously differentiable in $a = 0$. Lemma 1 below then shows that there is an $\tilde{a} > 0$ (which depends on θ) such that $\partial C(a,\theta)/\partial\theta > 0$ for all θ when $a \in (0, \tilde{a})$ if

$$P(\bar{I}+\theta) < 3R(\bar{I}+\theta) + \frac{\psi''(0)V'(\bar{I}+\theta)}{\sum_{i=1}^{n} \pi_i(0)\left(v_i'(0)\right)^2}, \tag{3}$$

and, conversely, if there is such an action $\tilde{a}$, then (3) holds with less than or equal to.

Now, the derivatives $v_i'(0)$ can be calculated by differentiating (2) and solving the corresponding linear system to obtain $v_i'(0) = \Gamma_i\psi''(0)$, where

$$\Gamma_i = \left.\frac{\sum_{j\neq i}\left(\pi_i\pi_j' - \pi_i'\pi_j\right)}{\pi_i\sum_{j<k}\frac{\left(\pi_k\pi_j' - \pi_k'\pi_j\right)^2}{\pi_j\pi_k}}\right|_{a=0} = \left.\frac{-\pi_i'}{\pi_i\sum_{j<k}\frac{\left(\pi_k\pi_j' - \pi_k'\pi_j\right)^2}{\pi_j\pi_k}}\right|_{a=0}.$$

By our assumptions on π', it follows that at least one Γ_i is not null.

To complete the proof, recall that $P(\hat{I}+\hat{\theta}) > 3R(\hat{I}+\hat{\theta})$ for $\hat{I}$ and $\hat{\theta}$, and we have set $\bar{I} = \hat{I}$ and $\theta = \hat{\theta}$. Then there exists a threshold $\tilde{k} > 0$ such that if $\psi''(0) > \tilde{k}$,

$$P(\hat{I}+\hat{\theta}) > 3R(\hat{I}+\hat{\theta}) + \frac{\psi''(0)V'(\bar{I}+\theta)}{\sum_{i=1}^{n} \pi_i(0)\left(v_i'(0)\right)^2} = 3R(\hat{I}+\hat{\theta}) + \frac{V'(\bar{I}+\theta)}{\psi''(0)\sum_{i=1}^{n} \pi_i(0)\,\Gamma_i^2},$$

and therefore $\partial C(\hat{a}, \hat{\theta})/\partial\theta < 0$ for some action $\hat{a} \in (0, \tilde{a})$. But then $\partial C(\hat{a}, \theta)/\partial\theta < 0$ for all levels of wealth θ in an open neighborhood of $\hat{\theta}$.

To finish the construction of a principal-agent problem where the principal prefers a richer agent, notice that MLRP, CDFC, and the assumption that for each a, $\pi_i'(a) \neq 0$ for some q_i, imply that $B_a(a) > 0$, and it is nonincreasing in a. Assume that $\pi(\cdot)$ is such that the cost function is convex in a (e.g., it follows from Jewitt, Kadan, and Swinkels (2008), adapted to a finite number of outcomes, that C is convex in a if $\pi(\cdot)$ is such that $L(a) = \min_{i^*} \sum_{i=1}^{i^*} \pi_i'(a) - t\sum_{i=1}^{i^*} \pi_i(a)$ is convex in a for all t). Then any action can be made optimal for the principal (i.e., solve $\max_{a\in A} B(a) - C(a,\theta)$) by a judicious choice of $(q_1, ..., q_n)$. In particular, this applies to any action in $(0, \tilde{a})$. Thus, there is an open interval of wealth levels θ for which the principal prefers a richer agent. □

Lemma 1 *Assume that $v_i(\cdot)$ is twice continuously differentiable in a, $i = 1, 2, ..., n$. If θ satisfies*

$$P(\bar{I}+\theta) < 3R(\bar{I}+\theta) + \frac{\psi''(0)V'(\bar{I}+\theta)}{\sum_{i=1}^{n} \pi_i(0)(v_i'(0))^2}, \tag{4}$$

then there exists an action $\tilde{a} > 0$ such that $\partial C(a, \theta)/\partial\theta > 0$ for all $a \in (0, \tilde{a})$. Conversely, if there is such an action $\tilde{a}$, then (4) holds with the weak inequality $\leq$.

Proof. Set $m(a) = \sum_{i=1}^{n}(\pi_i(a)g(v_i(a))$, and note that $m(0) = g(\overline{v})$. Thus, $\partial C(a, \theta)/\partial\theta$ is positive if and only if $m(a) > m(0)$.

Differentiating $m(a)$, we obtain $m'(a) = \sum_{i=1}^{n} \pi_i'(a)g(v_i(a)) + \sum_{i=1}^{n} \pi_i(a)g'(v_i(a))v_i'(a)$. Notice that $m'(0) = 0$. For $m'(0) = g(\bar{v})\sum_{i=1}^{n} \pi_i'(0) + g'(\bar{v})\sum_{i=1}^{n} \pi_i(0)v_i'(0)$, and $\sum_{i=1}^{n} \pi_i'(0) = 0$ while $\sum_{i=1}^{n} \pi_i(0)v_i'(0) = (\sum_{i=1}^{n} \pi_i(a)v_i(a))'|_{a=0} = (\bar{v}+\psi(a))'|_{a=0} = 0$, where the second equality follows from the participation constraint.

Thus, to assess the behavior of $m(a)$ near $a = 0$, we look at $m''(0)$. If it is positive at $a = 0$, then so it is (by continuity) in a right neighborhood of zero, i.e., for a small.

Differentiating $m'(a)$, we obtain after some algebra:

$$m''(0) = g'(\bar{v}) \left(2 \sum_{i=1}^{n} \pi_i'(0) v_i'(0) + \sum_{i=1}^{n} \pi_i(0) v_i''(0) \right) + g''(\bar{v}) \sum_{i=1}^{n} \pi_i(0) (v_i'(0))^2.$$

Differentiating $\sum_{i=1}^n \pi_i(a) v_i(a) = \bar{v} + \psi(a)$ yields $\sum_{i=1}^n \pi_i'(a) v_i(a) + \sum_{i=1}^n \pi_i(a) v_i'(a) = \psi'(a)$. Since $\sum_{i=1}^n \pi_i'(a) v_i(a) = \psi'(a)$, it follows that $\sum_{i=1}^n \pi_i(a) v_i'(a) = 0$, and its derivative yields $\sum_{i=1}^n \pi_i(a) v_i''(a) = -\sum_{i=1}^n \pi_i'(a) v_i'(a)$. Also, the derivative of the incentive constraint is $\sum_{i=1}^n \pi_i''(a) v_i(a) + \sum_{i=1}^n \pi_i'(a) v_i'(a) = \psi''(a)$, which converges to $\sum_{i=1}^n \pi_i'(0) v_i'(0) = \psi''(0)$ as a goes to zero since $v_i(a)$ converges to $\bar{v}$. Hence,

$$m''(0) = g'(\bar{v}) \left(\psi''(0) + \frac{g''(\bar{v})}{g'(\bar{v})} \sum_{i=1}^{n} \pi_i(0) (v_i'(0))^2 \right). \tag{5}$$

Since $\bar{v} = V(\bar{I} + \theta)$, (5) and the definition of $g(\cdot)$ imply that $m''(0) > 0$ if and only if (4) holds. Also, $m''(0) > 0$ implies that $m'(a) > m'(0) = 0$ for a near zero. Thus, there is an $\tilde{a} > 0$ such that $\partial C(a, \theta)/\partial \theta > 0$ for all $a \in (0, \tilde{a})$. To prove the converse, if (4) did not hold with $\leq$, then $m''(0) < 0$, and thus $\partial C(a, \theta)/\partial \theta < 0$ for a near zero. $\square$

5 Agent's Wealth and the Variance of the Contract

In Section 2.4 we asserted that an increase in θ increases the variance of the contract that implements any given action a above the lowest one. We now prove this assertion.

Let $I = (I_1, ..., I_n)$ be the contract that minimizes the cost of implementing action a. Since $E[I|a, \theta] = C(a, \theta)$, it follows that

$$\frac{\partial E[I|a, \theta]}{\partial \theta} = \sum_{i=1}^{n} \pi_i(a) \left(\frac{V'(\bar{I} + \theta)}{V'(I_i + \theta)} - 1 \right),$$

which we know can be positive or negative.

In turn, the variance of I is $\text{var}[I|a,\theta] = E[I^2|a,\theta] - (E[I|a,\theta])^2$, and thus

$$\begin{aligned}\frac{\partial \text{var}[I|a,\theta]}{\partial\theta} &= 2\sum_{i=1}^{n}\pi_i(a)I_i\frac{dI_i}{d\theta} - 2(E[I|a,\theta])\sum_{i=1}^{n}\pi_i(a)\frac{dI_i}{d\theta} \\ &= 2\sum_{i=1}^{n}\pi_i(a)\left(I_i - \sum_{i=1}^{n}\pi_i(a)I_i\right)\left(\frac{V'(\bar{I}+\theta)}{V'(I_i+\theta)} - 1\right), \qquad (6)\end{aligned}$$

where the last line follows by rearranging terms and using the expression above for the derivative of the mean of the contract.

To sign (6), rearrange the I_i's in increasing order and reinterpret the sum in (6) as already being reordered in this way. Then this is of the form $\sum_i \pi_i(a)f_i m_i$, with $f_i = I_i - E[I|a,\theta]$ and $m_i = (V'(\bar{I}+\theta)/V'(I_i+\theta)) - 1$. Now, f_i is increasing in i and crosses zero from negative to positive; moreover, $\sum_i \pi_i(a)f_i = 0$. Also, m_i is increasing in i. Hence, it follows from Lemma 1 in Persico (2000) that $\sum_i \pi_i(a)f_i m_i \geq 0$, thereby proving that the variance of the contract increases in θ.

6 Small Disutility of Effort

At the end of Section 3.2 we asserted that in the continuum of actions case, under some differentiability assumptions the principal prefers a poorer agent if the equilibrium disutility of effort is small. We now prove this assertion.

Let $a(\theta,\eta)$ be the optimal action implemented by the principal when the agent's wealth is θ and the disutility of effort parameter is η. Assume that $a(\theta,\cdot)$ is continuously differentiable in η for each θ, which holds if the contract that solves the cost minimization problem for each action is twice continuously differentiable (see Section 3.3 of this appendix for a rigorous justification). Let $\pi(\theta,\eta) \equiv B(a(\theta,\eta)) - C(a(\theta,\eta),\theta,\eta)$ the be principal's expected profit at the optimal contract. By the Envelope Theorem, $\partial\pi/\partial\theta = -\partial C(a(\theta,\eta),\theta,\eta)/\partial\theta$, and it easily follows that $\partial\pi/\partial\theta|_{\eta=0} = 0$. Thus, to prove the assertion it suffices to show that $\partial^2\pi/\partial\theta\partial\eta|_{\eta=0} < 0$. Now

$$\frac{\partial^2\pi}{\partial\theta\partial\eta}|_{\eta=0} = -\left(\frac{\partial^2 C(a(\theta,\eta),\theta,\eta)}{\partial\theta\partial a}\frac{\partial a(\theta,\eta)}{\partial\eta}\right)|_{\eta=0} - \frac{\partial^2 C(a(\theta,\eta),\theta,\eta)}{\partial\theta\partial\eta}|_{\eta=0}.$$

We know from the proof of Proposition 3 that the second term on the right side is negative. Regarding the first term, the derivative of the action converges to $\partial a(\theta, 0)/\partial \eta$, which is finite by continuous differentiability. Also,

$$\begin{aligned}\frac{\partial^2 C(a(\theta,\eta),\theta,\eta)}{\partial\theta\partial a}|_{\eta=0} &= \frac{1}{g(\bar{v})}\left(\sum_i \pi_i'(a)g(v_i)|_{\eta=0} + \sum_i \pi_i(a)g'(v_i)\frac{\partial v_i}{\partial a}|_{\eta=0}\right) \\ &= \frac{1}{g(\bar{v})}\left(g(\bar{v})\sum_i \pi_i'(a) + g'(\bar{v})\frac{\partial \sum_i \pi_i(a)v_i}{\partial a}|_{\eta=0}\right) \\ &= \frac{g'(\bar{v})}{g(\bar{v})}\eta\psi'(a)|_{\eta=0} = 0,\end{aligned}$$

where the last equality follows from $\sum_i \pi_i'(a) = 0$ and $\sum_i \pi_i(a)v_i = \bar{v} + \eta\psi(a)$. Thus, $\partial^2\pi/\partial\theta\partial\eta|_{\eta=0} < 0$, and this completes the proof of the assertion.

7 The Agent's Outside Option

In Section 3.4 we made two claims on the agent's outside option. We now prove them.

Consider first the case where the agent's outside option is $V(m+\theta)-\psi(\hat{a}) > V(\bar{I}+\theta)$, where $\hat{a}$ is the action implemented at the alternative job, and assume that the action the principal wants to implement is a.

Claim 1 *If $a < \hat{a}$ and $g(\cdot)$ is concave in v, then the principal prefers a richer agent.*

Proof. Let $\tilde{v} \equiv V(m+\theta)$ and consider *any* action $a < \hat{a}$. Proceeding as usual, the condition for $C(a,\cdot)$ to be decreasing in θ is $\sum_i \pi_i(a)g(v_i) \leq g(\tilde{v})$. Now

$$\begin{aligned}\sum_i \pi_i(a)g(v_i) &\leq g\left(\sum_i \pi_i(a)v_i\right) \\ &= g\left(\tilde{v} + \psi(a) - \psi(\hat{a})\right) \\ &\leq g\left(\tilde{v}\right),\end{aligned}$$

where the first inequality follows from $g(\cdot)$ concave, the equality from the binding participation constraint, and the second inequality from the premise that $\hat{a} > a$. This shows that $C(a,\theta)$ is decreasing in θ for any $a < \hat{a}$. Hence, if the principal optimally implements an action lower than $\hat{a}$, then he prefers a richer agent. □

Consider now the case where the agent's outside option is $\sum_j \hat{\pi}_j(\hat{a})V(\overline{I}_j+\theta)-\psi(\hat{a}) > V(\bar{I}+\theta)$.Assume that the action the principal wants to implement is a.

Claim 2 *If $a \geq \hat{a}$, $V(\cdot)$ satisfies DARA, and $g(\cdot)$ is convex in v, then the principal prefers a poorer agent.*

Proof. Let $\overline{v}_j \equiv V(\overline{I}_j + \theta)$ for all j, and consider any action $a \geq \hat{a}$. Proceeding as usual, the condition for $C(a, \cdot)$ to be increasing in θ is $\sum_i \pi_i(a) g(v_i) \geq \frac{1}{\sum_j \hat{\pi}_j(\hat{a}) \frac{1}{g(\overline{v}_j)}}$.

Notice that DARA is equivalent to $1/g(\cdot) = V'(h(\cdot))$ convex in v, for

$$\frac{d^2 V'(h(v))}{dv^2} = \frac{R}{V'}(P-R) \geq 0 \Leftrightarrow P \geq R,$$

which proves the assertion.

The convexity of $1/g$ (DARA) implies that

$$g\left(\sum_j \hat{\pi}_j(\hat{a})\overline{v}_j\right) \geq \frac{1}{\sum_j \hat{\pi}_j(\hat{a}) \frac{1}{g(\overline{v}_j)}}. \tag{7}$$

In turn, g convex implies

$$\sum_i \pi_i(a) g(v_i) \geq g\left(\sum_i \pi_i(a) v_i\right) \tag{8}$$

Finally, the binding participation constraint and $a \geq \hat{a}$ imply

$$g\left(\sum_i \pi_i v_i\right) = g\left(\sum_j \hat{\pi}_j \overline{v}_j + \psi(a) - \psi(\hat{a})\right) \geq g\left(\sum_j \hat{\pi}_j \overline{v}_j\right). \tag{9}$$

From (7)–(9) we obtain

$$\sum_i \pi_i(a) g(v_i) \geq \frac{1}{\sum_j \hat{\pi}_j(\hat{a}) \frac{1}{g(\overline{v}_j)}}$$

and thus $C(a, \cdot)$ increases in θ for any $a \geq \hat{a}$. Hence, if the principal optimally implements an action bigger than $\hat{a}$, then he prefers a poorer agent. $\square$

8 Large Wealth with $n = 2$

Recall the conditions we imposed in Section 3.4 of the paper:

(a) There is a threshold $\tilde{v}$ such that either $g(\cdot)$ is convex in v when $v \in (\tilde{v}, \infty)$, or $g(\cdot)$ is concave in v when $v \in (\tilde{v}, \infty)$ and $\lim_{v \to \infty} -g''(v)/g'(v) = 0$.

(b) For any $a \in A$ there is an optimal $(v_1, v_2, \ldots, v_n)$ with $|v_i - \bar{v}| \leq K_a$ for all i, where $K_a > 0$ is independent of $\bar{v}$.

(c) $\sup_{a \in A} K_a < \infty$.

(d) The principal's optimal action is bounded away from the lowest action for all θ.

Assume that condition (a) holds. We will show that if $n = 2$ (so there are two possible outcomes), and $\pi(\cdot)$ satisfy the strict MLRP and CDFC, then conditions (b)–(d) hold when the action set is finite or is an interval.

If $A = \{a_1, a_2, ..., a_m\}$, and the principal wants to implement $a_k > a_1$, then, under the general assumptions made in Section 2 plus MLRP and CDFC, only the incentive constraint corresponding to a_{k-1} binds (Grossman and Hart (1983) p. 34). Hence,

$$\begin{aligned} v_1 &= \bar{v} + \psi(a_k) - \pi_2(a_k)\frac{\psi(a_k) - \psi(a_{k-1})}{\pi_2(a_k) - \pi_2(a_{k-1})}, \\ v_2 &= \bar{v} + \psi(a_k) + (1 - \pi_2(a_k))\frac{\psi(a_k) - \psi(a_{k-1})}{\pi_2(a_k) - \pi_2(a_{k-1})}. \end{aligned}$$

Since $\pi_2(\cdot)$ is strictly increasing, it follows that $\pi_2\left(a_k\right) - \pi_2\left(a_{k-1}\right) > 0$. Thus, we can set

$$K_{a_k} = \psi(a_k) + \max\left\{\pi_2(a_k)\frac{\psi(a_k) - \psi(a_{k-1})}{\pi_2(a_k) - \pi_2(a_{k-1})}, (1 - \pi_2(a_k))\frac{\psi(a_k) - \psi(a_{k-1})}{\pi_2(a_k) - \pi_2(a_{k-1})}\right\} > 0.$$

Clearly, $\sup_{a_k \in A} K_{a_k} < \infty$, showing that (b)–(c) hold ((d) is not needed here).

Let now $A = [0, \bar{a}]$. The optimal contract that implements an action $a > 0$ is given

$$\begin{aligned} v_1 &= \overline{v} + \psi(a) - \pi_2(a)\frac{\psi'(a)}{\pi_2'(a)} \\ v_2 &= \overline{v} + \psi(a) + (1 - \pi_2(a))\frac{\psi'(a)}{\pi_2'(a)}. \end{aligned}$$

Therefore, we can set the value of K_a as

$$K_a = \psi(a) + \max\{\pi_2(a)(\psi'(a)/\pi_2'(a)), (1-\pi_2(a))(\psi'(a)/\pi_2'(a))\} > 0.$$

The continuity of the functions involved in the definition of K_a and the fact that $\pi_2'(a) > 0$, yield $\sup_{a\in A} K_a < \infty$. Hence, conditions (b)–(c) hold in this case as well.

Regarding condition (d), it holds if MLRP is strict, the vector of outcomes is large enough (which is a plausible assumption in, e.g., the CEO application), and there is an arbitrarily large upper bound on wealth. Alternatively, one can show that if $V(\cdot)$ is such that $g''(\cdot)$ is increasing in v, then condition (d) is *not* needed in this case ((a)–(c) suffice). We prove both assertions now:

Condition (d) from Primitives. Assume two outcomes, $\pi_2'(a) > 0$ for all a, and $\theta \in [0, \bar{\theta}]$ for $\bar{\theta} > 0$ arbitrarily large. We will show that if $\Delta q = q_2 - q_1$ is sufficiently large, then condition (d) holds (i.e., there is an action $\tilde{a}$ such that the principal's optimal action is greater than or equal to $\tilde{a}$ for all $\theta \in [0, \bar{\theta}]$).

Recall that the optimal contract that implements action a in this case is

$$\begin{aligned} v_1 &= \overline{v} + \psi(a) - \pi_2(a)\frac{\psi'(a)}{\pi_2'(a)} \\ v_2 &= \overline{v} + \psi(a) + (1-\pi_2(a))\frac{\psi'(a)}{\pi_2'(a)}, \end{aligned}$$

and thus the cost function is $C(a,\theta) = \pi_2(a)h(v_2) + (1-\pi_2(a))h(v_1) - \theta$, while the expected revenue is $B(a) = q_1 + \pi_2(a)\Delta q$. It is straightforward to show that $C_a(0,\theta) = 0$ and thus $B_a(0) - C_a(0,\theta) = \pi_2'(0)\Delta q > 0$. In general

$$B_a(a) - C_a(a,\theta) = \pi_2'(a)\Delta q - \pi_2'(a)\left(h(v_2) - h(v_1)\right) - \frac{\partial\left(\frac{\psi'(a)}{\pi_2'(a)}\right)}{\partial a}(1-\pi_2(a))\pi_2(a)\left(h'(v_2) - h'(v_1)\right).$$

Notice that $C_a(a,\theta)$ is continuous on $[0,\overline{a}] \times \left[0,\overline{\theta}\right]$. Let $\rho \in (0,\infty)$ be

$$\rho = \max_{[0,\overline{a}]\times\left[0,\overline{\theta}\right]} \pi_2'(a)\left(h(v_2) - h(v_1)\right) + \frac{\partial\left(\frac{\psi'(a)}{\pi_2'(a)}\right)}{\partial a}\pi_1(a)\pi_2(a)\left(h'(v_2) - h'(v_1)\right).$$

Take $\widetilde{a} \in (0, \overline{a}]$, and consider $\Delta q = q_2 - q_1 > 0$ large enough so that

$$\pi_2'(\widetilde{a})\,(q_2 - q_1) > \rho.$$

Then $B_a(a) - C_a(a, \theta) > 0$ for all $a \in (0, \tilde{a}]$ and $\theta \in [0, \bar{\theta}]$, and hence $\text{argmax}_{a \in [0,\bar{a}]} B(a) - C(a, \theta) \geq \tilde{a} > 0$ for all $\theta \in [0, \bar{\theta}]$, thereby proving that condition (d) holds.

Increasing $g''(\cdot)$. When $g''(\cdot)$ is increasing in v, the principal prefers a poorer agent in the two-outcome case without imposing condition (d). We show this by proving that $C(a, \cdot)$ is increasing in θ for all $a \in [0, \bar{a}]$ when θ is large enough. As a result, $\max_{a \in [0,\bar{a}]} B(a) - C(a, \theta)$ is decreasing in θ when θ is sufficiently large.

Assume $n = 2$, and recall that Section 3.3 assumes that $g(\cdot)$ is concave in v and $g''(v)/g'(v)$ converges to zero as v goes to infinity. We also assume here that $g''(\cdot)$ is increasing in v. The derivative of the cost function with respect to θ satisfies the following inequality:

$$\begin{aligned}
C_\theta(a, \theta) &= \frac{1}{g(\bar{v})}\left(\pi_2(a) g(v_2) + (1 - \pi_2(a)) g(v_1) - g(\bar{v})\right) \\
&= \frac{1}{g(\bar{v})}\left((\pi_2(a) g(v_2) + (1 - \pi_2(a)) g(v_1) - g(\bar{v} + \psi(a))) + (g(\bar{v} + \psi(a)) - g(\bar{v}))\right) \\
&\geq \frac{1}{g(\bar{v})}\left((\pi_2(a) g(v_2) + (1 - \pi_2(a)) g(v_1) - g(\bar{v} + \psi(a))) + g'(\bar{v} + \psi(a))\psi(a)\right),
\end{aligned}$$

where the inequality uses the concavity of g.

The assumptions on $g(\cdot)$ imply that $g''(v_1) \leq g''(v) \leq 0$, for all $v \geq v_1$.

Using a second order Taylor expansion around $\bar{v} + \psi(a)$, we obtain

$$\pi_2(a) g(v_2) + (1 - \pi_2(a)) g(v_1) - g(\bar{v} + \psi(a)) \geq 0.5 g''(v_1) \pi_2(a)(1 - \pi_2(a)) \frac{\psi'(a)^2}{\pi_2'(a)^2}.$$

Thus,

$$\begin{aligned}
C_\theta(a, \theta) &\geq \frac{\psi(a)}{g(\bar{v})}\left(0.5 g''(v_1) \frac{\pi_2(a)(1 - \pi_2(a))}{\pi_2'^2} \frac{\psi'(a)^2}{\psi(a)} + g'(\bar{v} + \psi(a))\right) \\
&\geq M \frac{g'(\bar{v} + \psi(\bar{a}))}{g(\bar{v})} \psi(a) \left(\frac{g''\left(\bar{v} + \psi(\bar{a}) - \pi_2(\bar{a}) \frac{\psi'(\bar{a})}{\pi_2'(\bar{a})}\right)}{g'(\bar{v} + \psi(\bar{a}))} + M^{-1}\right),
\end{aligned}$$

where $M = \max_{a\in[0,\bar{a}]} 0.5(\pi_2(a)(1-\pi_2(a))/\pi_2'(a)^2)(\psi'(a)^2/\psi(a)) \in (0,\infty)$. (Notice that since $\psi(0) = \psi'(0) = 0$, L' Hopital's Rule yields $\lim_{a\to 0}\psi'(a)^2/\psi(a) = 2\psi''(0) > 0$.)

Since $\lim_{v\to\infty} g''(v)/g('(v) = 0$, the first term in parenthesis goes to zero as $\bar{v}$ goes to infinity. Since $V(\cdot)$ is unbounded, it follows that there exists a threshold $\theta^* < \infty$ such that, if $\theta \geq \theta^*$, then $C_\theta(a,\theta) \geq 0$ for all a and hence the principal prefers a poorer agent.

9 Appendix: Continuity of the Optimal Contract

In this appendix we prove that the optimal contract is continuous in agent's wealth. The proof only requires that $V(\cdot)$, $\pi(\cdot)$, and $\psi(\cdot)$ be $\mathcal{C}^2$. We proceed in terms of the transformed variables $z_i = v_i - V(\bar{I},\theta)$, $i = 1,...,n$ defined above in Section 3.1. For definiteness, we will focus on the cost minimization problem for $A = [0,\overline{a}]$, for $a \in A, a > 0$, fixed. But it will be clear below that the *same* results hold in the finite case where the first-order condition of the agent's problem is replaced by a finite number of inequalities (see the parenthetical remark in the proof of Claim 2).

Consider

$$\begin{aligned} C(a,\theta) = \min \quad & \textstyle\sum_i \pi_i(a)\ h\left(z_i + V\left(\bar{I}+\theta\right)\right) \ -\theta \\ s.t. \quad & \textstyle\sum_i \pi_i(a)\ z_i = \psi(a)\,, \\ & \textstyle\sum_i \pi_i'(a)\ z_i = \psi'(a)\,. \end{aligned}$$

Set $f(z;\theta) = f(z_1,...z_n;\theta) := \sum_i \pi_i(a)\ h\left(z_i + V\left(\bar{I}+\theta\right)\right) - \theta$. If $z^*(\theta)$ denotes the unique solution of this problem, then $C(a,\theta) = f(z^*(\theta);\theta)$.

Claim 3 *The cost function $C(a,\theta)$ is upper semi-continuous on θ.*

Proof: Fix any θ^* and take the unique solution $z^*(\theta^*)$. Then, for any $\varepsilon > 0$, consider $\delta > 0$ such that

$$|f(z;\theta) - f(z^*(\theta^*);\theta^*)| < \varepsilon \quad if \quad \|(z,\theta) - (z^*(\theta^*),\theta^*)\| < \delta.$$

Observe that for all $\theta \in (\theta^* - \delta, \theta^* + \delta)$, we have

$$\|(z^*(\theta^*),\theta) - (z^*(\theta^*),\theta^*)\| = |\theta - \theta^*| < \delta,$$

so

$$\left|f\left(z^*\left(\theta^*\right);\theta\right)-f\left(z^*\left(\theta^*\right);\theta^*\right)\right|<\varepsilon,$$

consequently

$$f\left(z^*\left(\theta\right);\theta\right)\leq f\left(z^*\left(\theta^*\right);\theta\right)<f\left(z^*\left(\theta^*\right);\theta^*\right)+\varepsilon.$$

Hence

$$C\left(a,\theta\right)<C\left(a,\theta^*\right)+\varepsilon,$$

which gives

$$\limsup_{\theta\to\theta^*}C\left(a,\theta\right)\leq C\left(a,\theta^*\right)+\varepsilon$$

for all $\varepsilon>0$. Therefore

$$\limsup_{\theta\to\theta^*}C\left(a,\theta\right)\leq C\left(a,\theta^*\right).$$

□

Claim 4 *The optimal contract $z^*(\theta)$ is continuous.*[2]

Proof. Take any θ^* and any sequence $\left\{\theta^k\right\}$ converging to θ^*. Write $z^*=z^*\left(\theta^*\right)$ and $z^k=z^*\left(\theta^k\right)$, $k=1,2,\ldots$. We need to show that $z^k\to z^*$.

Observe that if $z^k\to\overline{z}$, then the continuity of f and Claim 1 give

$$f\left(\overline{z},\theta^*\right)=\lim_{k\to\infty}f\left(z^k;\theta^k\right)=\lim_{k\to\infty}C\left(a;\theta^k\right)\leq C\left(a;\theta^*\right)=f\left(z^*;\theta^*\right),$$

which implies that $\overline{z}=z^*=z^*\left(\theta^*\right)$ because of the uniqueness of the solution of the cost minimization problem. Hence it is enough to show that the sequence $\left\{z^k\right\}$ is bounded, because then any of its convergent subsequence has to converge to z^*.

Towards a contradiction, suppose that $\left\{z^k\right\}$ is unbounded, and assume (by passing to a subsequence if necessary) that $z^k\neq z^*$ and

$$\frac{z^k-z^*}{\left\|z^k-z^*\right\|}\to d,$$

for some vector $d\in\mathbb{R}^n$ with $\|d\|=1$. We will show that $z^*+d\in Z^*:=\left\{z^*\left(\theta^*\right)\right\}=\left\{z^*\right\}$, thereby reaching an obvious contradiction.

[2] This proof of this claim builds on (Gaya, Lopez, and Vera de Serio 2003) Lemma 3.6, Proposition 4.4, and Theorem 4.8; and (Goberna and Lopez 1998), Theorems 10.3 (ii)-(iii), 10.4 (i)-(ii).

Since z^* and all z^k satisfy the constraint restrictions, we have

$$0 = \frac{1}{\|z^k - z^*\|} \sum_i \pi_i(a) \ \left(z_i^k - z_i^*\right) = \pi(a) \cdot \frac{z^k - z^*}{\|z^k - z^*\|} \to \pi(a) \cdot d, \tag{10}$$

$$0 = \frac{1}{\|z^k - z^*\|} \sum_i \pi_i'(a) \ \left(z_i^k - z_i^*\right) = \pi'(a) \cdot \frac{z^k - z^*}{\|z^k - z^*\|} \to \pi'(a) \cdot d, \tag{11}$$

which implies that $z^* + d$ satisfies the participation constraint and the incentive constraint, i.e. it is an element of the feasible set of the cost minimization problem.

(In the finite case we have to consider the finite set of inequalities given by the incentive constraints. For any $a' \in A$,

$$\begin{aligned} \sum_i \left(\pi_i(a) - \pi_i(a')\right) \frac{\left(z_i^k - z_i^*\right)}{\|z^k - z^*\|} &= \frac{1}{\|z^k - z^*\|} \sum_i \left(\pi_i(a) - \pi_i(a')\right) \ z_i^k \\ &\quad - \sum_i \left(\pi_i(a) - \pi_i(a')\right) \ \frac{z_i^*}{\|z^k - z^*\|} \\ &\geq \frac{1}{\|z^k - z^*\|} \left(\psi(a) - \psi(a')\right) \\ &\quad - \sum_i \left(\pi_i(a) - \pi_i(a')\right) \ \frac{z_i^*}{\|z^k - z^*\|}. \end{aligned} \tag{12}$$

Thus, taking limit as $k \to \infty$, we obtain $\pi(a) \cdot d \geq \pi(a') \cdot d$. Hence $\pi(a) \cdot (z^* + d) - \psi(a) \geq \pi(a') \cdot (z^* + d) - \psi(a')$, and again $z^* + d$ is an element of the feasible set. This is the only significant modification needed in the finite case. The rest holds with minor changes.)

Now, let $c^* = C(a; \theta^*)$ and observe that

$$\begin{aligned} Z^* &= \left\{ z : \sum_i \pi_i(a) \ z_i = \psi(a), \sum_i \pi_i'(a) \ z_i = \psi'(a), \ f(z; \theta^*) \leq c^* \right\} \\ &= \left\{ z : \begin{array}{c} \sum_i \pi_i(a) \ z_i = \psi(a), \sum_i \pi_i'(a) \ z_i = \psi'(a); \\ u \cdot z \leq F(u; \theta^*) + c^*, \ u = \nabla_z f(z; \theta^*) \end{array} \right\}, \end{aligned} \tag{13}$$

where $F(u;\theta^*)$ is the Fenchel conjugate of $f(z;\theta^*)$, defined for any $u \in \mathbb{R}^n$ by

$$F(u;\theta^*) = \sup_{z\in\mathbb{R}^n} \{u \cdot z - f(z;\theta^*)\},$$

which in particular satisfies that $F(u;\theta^*) + f(z;\theta^*) = u \cdot z$ whenever $u = \nabla_z f(z;\theta^*)$.[3]

By inspection of (10), (11), and (13), it is apparent that $z^* + d$ would be a solution of the cost minimization problem, i.e. $z^* + d \in Z^*$, if $d \cdot \nabla_z f(z;\theta^*) \leq 0$ for all $z \in \mathbb{R}^n$.

Observe that the sequence $\left\{f\left(\cdot;\theta^k\right)\right\}$ converges uniformly to $f(\cdot;\theta^*)$ on compact sets.[4] Hence, we can apply Theorem 4.2 in (Attouch and Beer 1993). For any $\overline{z} \in \mathbb{R}^n$ and $\overline{u} = \nabla_z f(\overline{z};\theta^*)$, this Theorem gives the existence of sequences $\left\{u^k\right\}$ and $\left\{y^k\right\}$ in $\mathbb{R}^n$ such that $u^k = \nabla_z f\left(y^k;\theta^k\right)$, $u^k \to \overline{u}$, and $y^k \to \overline{z}$. It follows that

$$\begin{aligned}
& u^k \cdot \frac{z^k - z^*}{\|z^k - z^*\|} + u^k \cdot \frac{z^* - y^k}{\|z^k - z^*\|} \\
= \;& \frac{1}{\|z^k - z^*\|} u^k \cdot \left(z^k - y^k\right) \\
\leq \;& \frac{1}{\|z^k - z^*\|} \left(f\left(z^k;\theta^k\right) - f\left(y^k;\theta^k\right)\right) \\
= \;& \frac{1}{\|z^k - z^*\|} \left(C\left(a,\theta^k\right) - f\left(y^k;\theta^k\right)\right).
\end{aligned}$$

where the $\leq$ sign in the third line is justified by the convexity of the function $f\left(\cdot;\theta^k\right)$. Letting $k \to \infty$, the first and last lines yield

$$\overline{u} \cdot d \leq 0,$$

which holds because $u^k \cdot \frac{z^k - z^*}{\|z^k - z^*\|} \to \overline{u} \cdot d$, $\frac{1}{\|z^k - z^*\|} \to 0$, and all the other expressions are bounded since the sequence $\left(u^k, y^k, \theta^k\right)$ is convergent and $\limsup_{k\to\infty} C\left(a,\theta^k\right) \leq C(a,\theta^*) < \infty$. This completes the proof that $z^* + d \in Z^* = \{z^*(\theta^*)\} = \{z^*\}$. Since $\|d\| = 1$, we have reached the sought after contradiction.

[3] In general, $u \in \partial f(z;\theta^*)$, if f is not differentiable. See (Rockafellar 1970) Section 12 for further properties of the Fenchel conjugate of a convex function.

[4] We can apply Arzelà–Ascoli theorem to the convergent sequence $\left\{f\left(\cdot;\theta^k\right)\right\}$, which has uniformly bounded derivatives, besides being uniformly bounded itself on compact sets. To see this it is enough to consider the restriction of the continuously differentiable function f to compact sets of the form $B_1 \times B_2$, where B_1 is a compact box in $\mathbb{R}^n$ and B_2 is the compact set formed by θ^* and all θ^k, $k = 1, 2, \ldots$. Then, take bounds for f and $\|\nabla_z f\|$ on $B_1 \times B_2$.

Therefore $\{z^k\}$ is bounded and thus $z^*(\theta^k) = z^k \to z^* = z^*(\theta^*)$. □

Corollary: The set $Z^*([0,\overline{\theta}])$ is compact for any fixed $\overline{\theta} > 0$.

Proof: Since $Z^*([0,\overline{\theta}])$ is the image of a compact set through the continuous function $z^*(\theta)$, it follows that it is compact. □

Corollary: The cost function $C(a,\theta)$ is continuous on θ.

Proof: $C(a,\theta) = f(z^*(\theta);\theta)$ is a composition of continuous functions. □

Remark: We have kept the action a fixed to avoid complicating the notation. Notice, however, that we can repeat all the steps above considering, instead of $\{\theta^k\}$, a sequence $\{(a^k,\theta^k)\}$ converging to (a^*,θ^*). This way we would obtain the continuity of the optimal contract $z^*(a,\theta)$ and of the cost $C(a,\theta)$ as functions of the couple (a,θ). The only significant change needed is in obtaining (10) and (11) (and the incentive constraints in the finite case), e.g.

$$\begin{aligned}\frac{\psi(a^k)-\psi(a^*)}{\|z^k-z^*\|} &= \frac{1}{\|z^k-z^*\|}\left(\sum_i \pi_i(a^k)\ z_i^k - \sum_i \pi_i(a^*)\ z_i^*\right) \\ &= \frac{1}{\|z^k-z^*\|}\left(\pi(a^k)\cdot(z^k-z^*) + \left(\pi(a^k)-\pi(a^*)\right)\cdot z^*\right) \\ &= \pi(a^k)\cdot\frac{z^k-z^*}{\|z^k-z^*\|} + \frac{\left(\pi(a^k)-\pi(a^*)\right)\cdot z^*}{\|z^k-z^*\|};\end{aligned}$$

letting $k\to\infty$ we get $0 = \pi(a^*)\cdot d$, by the C^1 property of the functions ψ and π (in the finite case, consider $\sum_i \left(\pi_i(a^k) - \pi_i(a')\right)\frac{(z_i^k - z_i^*)}{\|z^k-z^*\|}$ in (12)).

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