



Available online at [www.sciencedirect.com](http://www.sciencedirect.com)



Review of Economic Dynamics 8 (2005) 565–599

Review of  
Economic  
Dynamics

[www.elsevier.com/locate/red](http://www.elsevier.com/locate/red)

## Income taxation and marital decisions

Hector Chade <sup>a</sup>, Gustavo Ventura <sup>b,\*</sup>

<sup>a</sup> Department of Economics, Arizona State University, Main Campus, PO Box 873806, Tempe, AZ 85287-3806, USA

<sup>b</sup> Department of Economics, Pennsylvania State University, 613 Kern Bldg., University Park, PA 16803, USA

Received 6 August 2002; revised 13 January 2005

Available online 8 March 2005

---

### Abstract

Differential tax treatment of married and single people is a key feature of the tax code in the US and other countries. We analyze its equilibrium and welfare effects in a matching model with search frictions and nontransferable utility. We find that an increase in taxes on married people unambiguously reduces the equilibrium number of marriages, but it need not make both men and women more reluctant to marry. We also show that it is optimal to give married couples a preferential tax treatment. A quantitative analysis using US data reveals that relatively large changes in taxes are associated with small changes in the number of marriages and divorces. Finally, we extend the model to allow for cohabitation as an alternative to marriage, and find that the number of marriages becomes more sensitive to increases in the marriage tax penalty. The magnitude of the resulting changes, however, is still moderate.

© 2005 Elsevier Inc. All rights reserved.

*JEL classification:* H2; D1

*Keywords:* Marriage penalty; Marriage tax; Two-sided search; Marriage; Cohabitation

---

\* Corresponding author.

E-mail addresses: [hector.chade@asu.edu](mailto:hector.chade@asu.edu) (H. Chade), [cjv10@psu.edu](mailto:cjv10@psu.edu) (G. Ventura).

## 1. Introduction

The income tax treatment of married and single individuals varies markedly across countries and jurisdictions. In several places the prevailing tax laws are significantly non-neutral with respect to marital status. The US constitutes a clear example of non-neutrality: married couples pay federal taxes based on their joint income and face a tax schedule that differs from the one applied to two single individuals. As a result, the combined tax liabilities of two individuals can drastically change if they get married. This feature of the tax code can generate a tax ‘penalty’ (marriage tax) or a tax ‘bonus’ (marriage subsidy) associated with marriage, as well as variations in marginal tax rates. Other leading examples of countries where the tax code lacks marriage neutrality are Germany and France.<sup>1</sup>

Little modeling effort has been devoted to the analysis of marital behavior under differential tax treatment of married and single people, even though the issue is not new and is the subject of numerous policy debates. This paper constitutes a first step to fill this void. We study the equilibrium and welfare effects of income taxation on marital decisions in a marriage market model with costly search for potential marriage partners and nontransferable utility. The model is based on the two-sided search framework developed by Burdett and Wright (1998), suitably modified to account for the relevant features of the problem under analysis.

In the model, *ex ante* identical single men and women randomly meet pairwise and, after observing an idiosyncratic taste parameter that reflects each agent’s preference for the current potential partner, they get married if both find the current partner acceptable. Otherwise, they go back to the pool of singles and wait for the next meeting. Agents remain married until one of the spouses dies or decides to dissolve the match. When single, agents consume the income they generate, while married people divide the income of the household using a fixed sharing rule. Income taxation depends on marital status, i.e., the tax rate levied on singles need not be the same as the one levied on married people. Thus, taxes affect agents’ optimal marital decisions and hence the equilibrium number of marriages.

We first focus on the equilibrium effects of changes in the differential tax treatment of married and single individuals. We find that an increase in the marriage tax has two effects on the number of marriages. On the one hand, each individual becomes more selective in their acceptance decision of potential mates, since the income gains from marriage decrease. This effect has a clear negative impact on the number of marriages. On the other hand, there is an indirect or two-sided search effect that mitigates the initial impact: agents realize that they are accepted less often and this makes them less selective in their marriage decisions. We prove that this effect, which is due exclusively to the existence of

---

<sup>1</sup> The German tax system is similar to the one in the US. A different scheme exists in France, where the taxable income of all family members is averaged out, and tax liabilities are calculated using the same schedule applied to single individuals. This implies that married couples in France typically experience a tax-induced marriage bonus. In countries such as Canada and Sweden, marriage is neither penalized nor favored by the tax law, as the unit subject to taxation is the individual rather than the couple or the family. In the US, the income tax law treats married and single people differently since 1948. It seems that differential taxation of married and single individuals goes back at least to the times of the Roman Emperor Augustus who, in order to foster family formation, introduced a legislation that set implicit taxes on unmarried adults, including widows and widowers (see Southern, 1998).

search frictions and to the fact that search is two-sided, can dominate the first effect for men or women, but not for both. In other words, an increase in the marriage tax need not make everybody more reluctant to get married. Although the net effect on the number of marriages is still negative, it is smaller than when the two-sided search effect is ignored, as it would be in a partial equilibrium model in which agents on one side of the market ignore the feedback effects of the strategies of agents on the other side, or in models without search frictions.

We then turn to the normative aspects of the problem. More precisely, we study the planner's problem of selecting the tax rates that maximize the expected discounted welfare of a cohort of men and women that enters the marriage market in any given period, subject to a balanced budget constraint for each cohort. We find that the optimal solution unambiguously requires a preferential tax treatment of married couples (i.e., a tax-induced marriage bonus). The intuition of this finding is as follows. In a matching equilibrium, individuals ignore the effects of their marriage decisions on the welfare of their prospective partners. As a result, the equilibrium is inefficient as some matching rents are left unexploited, which leads to a lower than optimal number of marriages. We show that the size of the optimal tax-induced marriage bonus is the one needed to increase the number of marriages to its efficient level.

To assess the quantitative implications of the analysis, we compute the model using US data. We find that relatively large increases in the marriage tax penalty have small effects on the number of marriages, and that the number of divorces can actually increase when the marriage tax penalty decreases. We also find that the optimal tax treatment of married and single people involves a rather large tax-induced marriage bonus.

Finally, we extend the model to allow for the (empirically relevant) possibility of cohabitation as an alternative to marriage. In the model, as it occurs in practice, agents who cohabit pay taxes as single individuals. We assume that cohabitation provides the same match benefits of marriage, but it is a more unstable arrangement. In equilibrium, cohabitation coexists with marriage, and agents transit from being single to being married or to cohabiting and vice versa, and also from cohabitation to marriage. To understand how previous results change in the presence of cohabitation, we calibrate the parameters of the model taking into account observations pertaining to both cohabitation and marriage. We find that the equilibrium number of marriages becomes more sensitive to changes in differential tax treatment than in the benchmark case. Intuitively, as this tax rate goes up, agents who were marginally inclined to marry prefer to cohabit instead. Hence, the number of agents cohabiting increases. Despite this increase, we find that the total number of individuals in a match (i.e., married plus cohabiting) decreases with an increase in the tax rate on married couples.

To the best of our knowledge, this is the first paper to provide an analytical characterization of the equilibrium and welfare effects of income taxation on marital decisions, and to allow for the possibility of cohabitation as an alternative to marriage. Our analysis is based on a model that lacks some important features of reality such as labor supply and fertility decisions, *ex ante* heterogeneous individuals in each population, progressive taxes, or a more sophisticated household decision problem. In spite of these omissions, we view our results as a useful starting point towards a thorough understanding of the marriage market effects of differential tax treatment of married and single individuals.

### *1.1. Related literature*

Using reduced-form regression models, several papers have attempted to estimate empirically the effects that income taxation has on marital decisions in the United States. Alm and Whittington (1995a, 1995b) constructed aggregate estimates of the marriage tax penalty, and regressed the total number of marriages on this variable and a set of covariates. Although they found that changes in the marriage tax penalty have a negative impact on the number of marriages, the magnitude of the effect is systematically small in all of their estimations. Sjoquist and Walker (1995) conducted a similar exercise using instead the marriage rate (a flow variable) as a dependent variable, but they found no statistically significant effect. Using panel data from PSID, Whittington and Alm (1996) estimated a discrete-time hazard model of the probability of divorce. They found that changes in the marriage tax penalty have small, but positive effect on the probability of divorce from first marriage.

We departed from the reduced-form tradition in a previous paper (Chade and Ventura, 2002), where we studied quantitatively the effects of tax reforms that partially, or completely, eliminate differential tax treatment in the US case. For that purpose, we built an equilibrium model of marriage, divorce and labor supply with heterogeneous agents. We found that tax reforms lead to relatively small increases in the number of marriages in general, but can imply substantial increases in the degree of assortative matching and in labor supply of married females.

Albeit rich and useful for a quantitative study, the model of that paper is too complex to be analytically tractable. In the current paper we cast the problem at hand in a simpler framework. As a result, this allows us to provide the first characterization of the positive and normative effects of the differential tax treatment of married and single people that we are aware of.

The literature on equilibrium matching models includes not only Burdett and Wright (1998) but also the models with ex ante heterogeneous agents by Burdett and Coles (1997), Chade (2001, 2002), Eeckhout (1999), Morgan (1996), Lu and Mc-Afee (1996), Bloch and Ryder (2000), Shimer and Smith (2000), and Smith (1997), among others (see Burdett and Coles, 1999 for a survey of the literature). Unlike these papers we only allow for ex post heterogeneity, a major simplification that accounts for the tractability of the model.

The paper is organized as follows. Section 2 describes the model. In Section 3 we analyze the equilibrium effects of a change in the differential tax treatment of married and single individuals. In Section 4 we analyze the welfare effects of differential tax treatment. Section 5 extends the model by allowing for endogenous match dissolutions. Section 6 presents a numerical example. Section 7 introduces cohabitation in the model, and Section 8 concludes. Appendix A contains most of the proofs.

## **2. Theoretical framework**

We analyze a marriage market with search frictions and differential tax treatment of married and single individuals. The basic model builds on the two-sided search framework developed in Burdett and Wright (1998).

Consider a stationary economy populated by a continuum of agents who live in continuous time and are of two types:  $m$  (males) and  $f$  (females). The measure of each population is for simplicity normalized to one. Each agent engages in the time consuming process of looking for a mate. Ex ante, individuals in each population are identical. At each meeting, a man observes a realization of a match-specific random variable  $\theta_m$ , distributed according to  $G_m(\theta_m)$ , and a woman observes realization of  $\theta_f$ , distributed according to  $G_f(\theta_f)$ . For simplicity, it is assumed that  $\theta_m$  and  $\theta_f$  are independently distributed on  $[0, \bar{\theta}]$ ,  $\bar{\theta} \leq \infty$ , and that  $G_f$  and  $G_m$  are differentiable. The corresponding densities are denoted by  $g_f(\theta_f)$  and  $g_m(\theta_m)$ , and are assumed to be continuous. These match-specific components are payoff-relevant; consequently, agents are homogeneous ex ante but heterogeneous ex post.

There is a meeting technology that yields the number of meetings among men and women as a function of the measure of unmatched individuals. This technology exhibits constant returns to scale; i.e., the total number of meetings per unit of time is  $N = \beta(1 - M)$ , where  $\beta$  is the contact rate for an individual and  $(1 - M)$  is the number of singles in the population (equal to the size of the population, minus the measure  $M$  of married agents).

Agents discount the future at the rate  $r$ . When single, the instantaneous utility of an agent is equal to his or her after-tax income  $w_i(1 - t^S)$ ,  $i = f, m$ , where  $t^S$  is the tax rate for a single person, and  $w_i$  is the instantaneous wage of an agent of type  $i$ . Wages are taken as given in the model in order to focus on marriage market decisions. If a couple decides to get married after observing  $\theta_m$  and  $\theta_f$ , then his instantaneous utility is  $k(1 - t^M)(w_m + w_f) + \theta_m$  and hers is  $(1 - k)(1 - t^M)(w_m + w_f) + \theta_f$ , where  $t^M$  is the tax rate for a married couple, and  $(k, 1 - k)$  are the shares of the total income that each spouse receives, which are taken as given in the model.<sup>2</sup>

Since the division of a couple's after-tax income is exogenously given and the match specific components are not shared by the spouses, this is a model with non-transferable utility (see Burdett and Wright, 1998 and Burdett and Coles, 1999 for a precise definition). This assumption has been widely used in the literature on marriage markets (see the discussion in Becker, 1973, pp. 834–836), and it intends to capture the idea that, in marriage, there are some limits on the utility that can be transferred between spouses.

Regarding taxes, we say that there is differential tax treatment between married and single individuals if  $t^M \neq t^S$ . If  $t^M - t^S > 0$ , then there exists a ‘marriage tax or penalty,’ while there is a ‘marriage subsidy or bonus’ if  $t^M - t^S < 0$ .

Consider the decision problem faced by a man. Meetings arrive according to a Poisson process with parameter  $\beta$ . In a meeting with a female, a man observes a realization of  $\theta_m$  and then decides whether to accept or reject the match. Obviously, the woman is facing an analogous problem, and the match is formed if and only if both find it acceptable. A married man does not generate any new marriage offer, and he is abandoned by his wife according to a Poisson process with parameter  $\lambda_m$ . Also, he dies according to another independent Poisson process with parameter  $\delta_m$ .

---

<sup>2</sup> When consumption is a public good within the household, then the instantaneous utility of a married agent  $i$  becomes  $v(1 - t^M)(w_m + w_f) + \theta_i$ , where  $v \in (0, 1]$ . Most of the results of the paper hold for this case as well.

Let  $U_m$  be the expected discounted utility of a single man, and let  $V_m(\theta_m)$  be the expected discounted utility of a man who is in a marriage characterized by a match specific component  $\theta_m$ . Formally, they are recursively defined as follows:

$$(r + \delta_m)U_m = (1 - t^S)w_m + \beta E[\max\{\xi_m(V_m(\theta_m) - U_m), 0\}], \quad (1)$$

$$(r + \delta_m)V_m(\theta_m) = k(1 - t^M)(w_m + w_f) + \theta_m + \lambda_m(U_m - V_m(\theta_m)), \quad (2)$$

where  $\xi_m$  is the probability that he is accepted in any given meeting. Equation (2) is equal to

$$V_m(\theta_m) = \frac{k(1 - t^M)(w_m + w_f) + \theta_m + \lambda_m U_m}{r + \delta_m + \lambda_m}. \quad (3)$$

An optimal strategy for a man is to accept a match if and only if the match specific component is above a threshold  $\theta_m^*$ , defined by  $V_m(\theta_m^*) = U_m$ . Substituting this in (3) yields

$$(r + \delta_m)U_m = \theta_m^* + k(1 - t^M)(w_m + w_f),$$

and we can rewrite (1) as follows:

$$\theta_m^* + k(1 - t^M)(w_m + w_f) - (1 - t^S)w_m = \beta \xi_m \int_{\theta_m^*}^{\bar{\theta}} (V_m(\theta_m) - V_m(\theta_m^*)) dG_m(\theta_m).$$

Integrating the right side by parts, using the derivative of  $V_m(\theta_m)$  with respect to  $\theta_m$  and the Fundamental Theorem of Calculus yields

$$\theta_m^* + k(1 - t^M)(w_m + w_f) - (1 - t^S)w_m = \frac{\beta \xi_m}{r + \delta_m + \lambda_m} \mu_m(\theta_m^*),$$

where  $\mu_m(\theta_m^*) \equiv \int_{\theta_m^*}^{\bar{\theta}} (1 - G_m(\theta_m)) d\theta_m$ .

Women face an analogous problem, and their (common) threshold  $\theta_f^*$  is implicitly defined by:

$$\theta_f^* + (1 - k)(1 - t^M)(w_m + w_f) - (1 - t^S)w_f = \frac{\beta \xi_f}{r + \delta_f + \lambda_f} \mu_f(\theta_f^*),$$

where  $\mu_f(\theta_f^*) \equiv \int_{\theta_f^*}^{\bar{\theta}} (1 - G_f(\theta_f)) d\theta_f$ .

When a man or a woman dies, he or she is replaced by a new entrant; if that individual was married, then the widowed agent goes back to the pool of singles. In this way, the size of each population remains constant.

A meeting generates a marriage proposal for a man if  $\theta_f \geq \theta_f^*$ . Hence, in equilibrium

$$\xi_m = (1 - G_f(\theta_f^*)) = -\mu'_f(\theta_f^*),$$

and similarly

$$\xi_f = (1 - G_m(\theta_m^*)) = -\mu'_m(\theta_m^*).$$

Since in this simple setting terminations only occur when one of the spouses dies, it follows that  $\lambda_f = \delta_m$  and  $\lambda_m = \delta_f$ .

**Definition 1.** A matching equilibrium is a pair  $(\theta_m^*, \theta_f^*)$  that satisfies the following equations:

$$\theta_m^* + k(1 - t^M)(w_m + w_f) - (1 - t^S)w_m = -\pi\mu'_f(\theta_f^*)\mu_m(\theta_m^*), \quad (4)$$

$$\theta_f^* + (1 - k)(1 - t^M)(w_m + w_f) - (1 - t^S)w_f = -\pi\mu'_m(\theta_m^*)\mu_f(\theta_f^*), \quad (5)$$

where  $\pi = \beta/(r + \delta_f + \delta_m)$ .

Equations (4)–(5) define a pair of downward sloping best response functions, one for males and one for females. In principle, they could cross more than once, giving rise to multiple equilibria.

Adapting Proposition 1 in Burdett and Wright (1998) to our model, yields the following existence and uniqueness result:

**Proposition 1.** Suppose (a)  $\mu_m$  and  $\mu_f$  are log-concave functions (i.e.,  $\mu''_j\mu_j - (\mu'_j)^2 \leq 0$ ,  $j = m, f$ ), and

$$(b) \quad E[\theta_m] > \frac{k(1 - t^M)(w_f + w_m) - (1 - t^S)w_m}{\pi} \quad \text{and}$$

$$E[\theta_f] > \frac{(1 - k)(1 - t^M)(w_f + w_m) - (1 - t^S)w_f}{\pi}.$$

Then there is a unique matching equilibrium with either  $\theta_f^* > 0$  or  $\theta_m^* > 0$  or both.

Henceforth, we will assume that  $\mu_m$  and  $\mu_f$  are log-concave and that condition (b), whose only role is to ensure that at least one of the thresholds is positive, holds. A sufficient condition for  $\mu_m$  and  $\mu_f$  to be log-concave is that  $g_m$  and  $g_f$  be log-concave. This is a relatively weak assumption, and it is satisfied by many standard density functions, including the uniform, exponential, normal, logistic, and chi-squared.

In a matching equilibrium, the flow into the pool of married agents in each population, given by  $(1 - M)\beta(1 - G_m(\theta_m^*))(1 - G_f(\theta_f^*))$  (number of meetings times the probability of marriage), must be equal to the flow out of this pool, given by  $M(\delta_f + \delta_m)$  (number of married agents times the sum of the probability of becoming a widow or widower plus the probability of dying).

The equilibrium measure of married agents in each population,  $M^*$ , is thus given by

$$\begin{aligned} M^* &= \frac{\beta(1 - G_m(\theta_m^*))(1 - G_f(\theta_f^*))}{\beta(1 - G_m(\theta_m^*))(1 - G_f(\theta_f^*)) + (\delta_f + \delta_m)} \\ &= \frac{\beta\mu'_m(\theta_m^*)\mu'_f(\theta_f^*)}{\beta\mu'_m(\theta_m^*)\mu'_f(\theta_f^*) + (\delta_f + \delta_m)} \\ &= \frac{\gamma}{\gamma + (\delta_f + \delta_m)}, \end{aligned} \quad (6)$$

where  $\mu'_i = -(1 - G_i)$ ,  $i = m, f$ , and  $\gamma = \beta\mu'_m\mu'_f$  (the arguments of  $\mu'_i$ ,  $i = m, f$ , and  $\gamma$  are omitted to simplify the notation).

### 3. Changes in differential tax treatment: equilibrium effects

We first investigate the equilibrium effects of changes in the differential tax treatment of married and single individuals. Although this may be due to a change in  $t^M$  or  $t^S$ , we mainly deal with the case in which only  $t^M$  changes and address the following questions: Do both men and women become more reluctant to marry when  $t^M$  increases? How does the number of married people change in this case?

Notice that  $M^*$  depends on  $t^M$  through the thresholds  $(\theta_f^*, \theta_m^*)$ . From (6), it follows that the sign of  $\partial M^*/\partial t^M$  is equal to the sign of  $\partial\gamma/\partial t^M$ , given by

$$\frac{\partial\gamma}{\partial t^M} = \beta \left( \mu_m'' \mu_f' \frac{\partial\theta_m^*}{\partial t^M} + \mu_f'' \mu_m' \frac{\partial\theta_f^*}{\partial t^M} \right). \quad (7)$$

The derivatives  $\partial\theta_m^*/\partial t^M$  and  $\partial\theta_f^*/\partial t^M$  can be found by implicit differentiation of (4) and (5). They are

$$\frac{\partial\theta_m^*}{\partial t^M} = \frac{k(1 + \pi\mu_f'\mu_m') - (1 - k)\pi\mu_m\mu_f''}{(1 + \pi\mu_f'\mu_m')^2 - \pi^2\mu_f''\mu_m\mu_m''\mu_f} (w_m + w_f), \quad (8)$$

$$\frac{\partial\theta_f^*}{\partial t^M} = \frac{(1 - k)(1 + \pi\mu_f'\mu_m') - k\pi\mu_f\mu_m''}{(1 + \pi\mu_f'\mu_m')^2 - \pi^2\mu_f''\mu_m\mu_m''\mu_f} (w_m + w_f). \quad (9)$$

Consider first the symmetric case in which  $G_f = G_m = G$ ,  $w_f = w_m = w$ ,  $k = 1/2$ , and  $\delta_f = \delta_m$ . The matching equilibrium is symmetric with both populations choosing the same threshold  $\theta^*$ , and thus  $\mu_f = \mu_m = \mu$ . In this case, the sign of  $\partial\theta^*/\partial t^M$  depends on the sign of the following expression:

$$\frac{1 + \pi(\mu'^2 - \mu''\mu)}{(1 + \pi\mu'^2)^2 - (\pi\mu''\mu)^2} = \frac{1}{1 + \pi\mu'^2 + \pi\mu''\mu} > 0. \quad (10)$$

Therefore,  $\theta^*$  increases when  $t^M$  increases, and it follows from (7) that  $M^*$  decreases. The intuition is rather simple: an increase in  $t^M$  reduces the ‘income gains’ from marriage, so agents demand higher match-specific components to marry.

It is easy to show that the resulting changes in  $\theta^*$  and in  $M^*$  are smaller than in the ‘one-sided’ case in which agents ignore the reaction of the other population: in that case, the analogue of (10) is  $1/(1 + \pi\mu'^2)$ . The intuition is the following: when  $t^M$  increases, there are two opposite effects on  $\theta^*$ . First, there is a *direct* effect: the decrease in the income gains from marriage increases the acceptance thresholds of men and women. Second, there is an *indirect* effect, which we call the *two-sided search* effect: each agent now faces a ‘tighter’ search environment (since they are accepted less often) and this makes them less selective. In the symmetric case, we have just showed that the first effect dominates for both men and women, and the net result is an increase in the thresholds. The two-sided search effect, however, makes the thresholds and the fraction of married people *less* sensitive to changes in the marriage tax than in the one-sided search case.

Things are a bit more complicated in the asymmetric case. Given the log-concavity assumption, the denominators of (8) and (9) are positive, for

$$\begin{aligned}
(1 + \pi \mu'_f \mu'_m)^2 - \pi^2 \mu''_f \mu_m \mu''_m \mu_f &= 1 + 2\pi \mu'_f \mu'_m + \pi^2 ((\mu'_f \mu'_m)^2 - \mu''_f \mu_m \mu''_m \mu_f) \\
&\geq 1 + 2\pi \mu'_f \mu'_m + \pi^2 ((\mu'_f \mu'_m)^2 - (\mu'_f \mu'_m)^2) \\
&= 1 + 2\pi \mu'_f \mu'_m > 0.
\end{aligned}$$

The numerators, however, can be positive or negative. As the next result shows, an increase in  $t^M$  can make one of the populations *more* selective and the other *less* selective in their acceptance decisions.

**Proposition 2.** *If  $t^M$  increases, then either*

- (i) *both  $\theta_m^*$  and  $\theta_f^*$  increase, or*
- (ii)  *$\theta_m^*$  increases and  $\theta_f^*$  decreases, or*
- (iii)  *$\theta_m^*$  decreases and  $\theta_f^*$  increases.*

*Case (ii) occurs if  $k$  is sufficiently close to one, while (iii) arises if  $k$  is sufficiently close to zero.*

Notice that an increase in the marriage tax need not make both men and women less inclined to marry. If the division of the household's income is sufficiently asymmetric, then the two-sided search effect dominates for one of the populations, leading to a *decrease* in the acceptance threshold. To understand the intuition of this result consider case (ii); since the male partner captures most of the household's income, an increase in  $t^M$  has a substantial direct effect on the acceptance threshold of men and a small direct effect on the women's threshold. But this implies that the two-sided search effect will be small for men and large for women. Case (ii) shows that this indirect effect outweighs the direct one for women.<sup>3</sup>

In Fig. 1, we illustrate cases (i) and (ii) when  $t^M$  increases from  $t_0^M$  to  $t_1^M$ . The functions  $r_m(\cdot)$  and  $r_f(\cdot)$  are the best response functions of males and females derived from (4) and (5). In the figure, the original intersection of the best response function is given by A. Case (i) is represented by a movement from A to C. Case (ii) is represented by a movement from A to B.

Proposition 2 also suggests the theoretical possibility that  $M^*$  could actually increase with an increase in  $t^M$ . Notice, however, that if we insert (8) and (9) into (7) then it is easy to show that a sufficient condition for  $\partial \gamma / \partial t^M$  to be nonpositive is

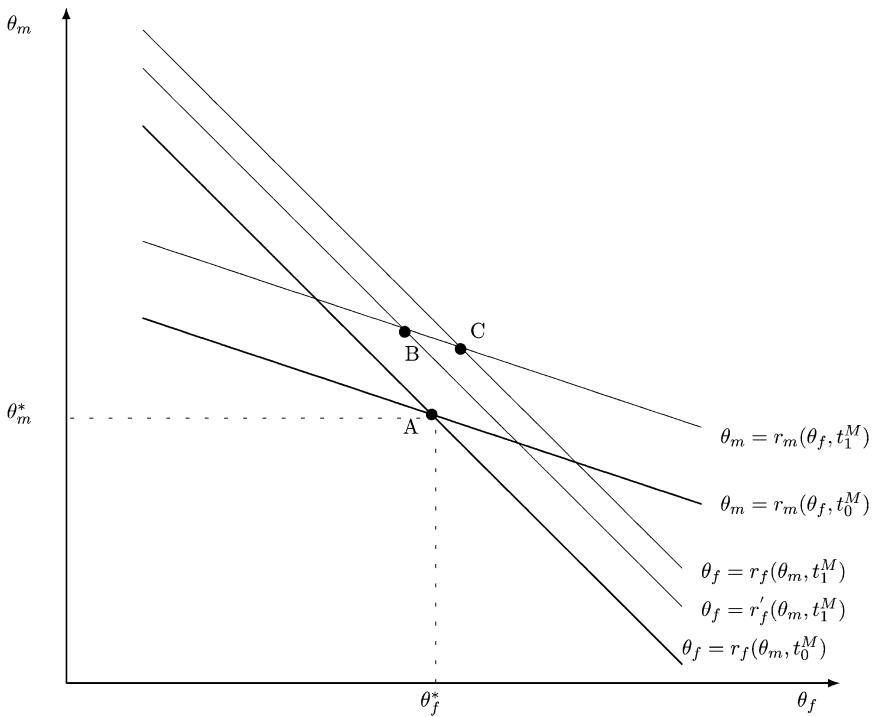
$$k \mu''_m (\mu'_f)^2 \mu'_m - (1-k) \mu_m \mu''_f \mu''_m \mu'_f + (1-k) \mu''_f (\mu'_m)^2 \mu'_f - k \mu_f \mu''_f \mu''_m \mu'_m \leq 0.$$

Under log-concavity, the left side of this expression is less than or equal to

$$k \mu''_m (\mu'_f)^2 \mu'_m - (1-k) (\mu'_m)^2 \mu''_f \mu'_f + (1-k) \mu''_f (\mu'_m)^2 \mu'_f - k (\mu'_f)^2 \mu''_m \mu'_m = 0.$$

Hence,  $M^*$  decreases when  $t^M$  increases.

<sup>3</sup> The proof uses a limiting argument as  $k$  goes to zero or one to show that the thresholds can move in opposite directions, but this is not the only 'asymmetry' that can lead to this result. For instance, the same happens if we change  $t^S$  instead, and  $w_m/w_f$  goes to zero or infinity. See Eqs. (11)–(12).

Fig. 1. Changes in  $\theta_f^*$  and  $\theta_m^*$ .

It is important to emphasize that the two-sided search effect is due *exclusively* to the existence of search frictions (i.e., search is a time consuming process and time is valuable). Indeed, as search frictions vanish (i.e.,  $r$  and  $\delta_i$ ,  $i = m, f$ , go zero) both thresholds converge to  $\bar{\theta}$ ; in the limit, agents would ‘sample’ marriage proposals until they observe  $\bar{\theta}$ , thereby rendering the two-sided search effect of a change in  $t^M$  irrelevant.

So far we have focused on the case in which  $t^M$  changes keeping  $t^S$  constant. Analogous results hold if  $t^S$  changes instead. Straightforward differentiation of (4)–(5) yields

$$\frac{\partial \theta_m^*}{\partial t^S} = -\frac{w_m(1 + \pi \mu'_f \mu'_m) - w_f \pi \mu_m \mu''_f}{(1 + \pi \mu'_f \mu'_m)^2 - \pi^2 \mu''_f \mu_m \mu''_m \mu_f}, \quad (11)$$

$$\frac{\partial \theta_f^*}{\partial t^S} = -\frac{w_f(1 + \pi \mu'_f \mu'_m) - w_m \pi \mu_f \mu''_m}{(1 + \pi \mu'_f \mu'_m)^2 - \pi^2 \mu''_f \mu_m \mu''_m \mu_f}. \quad (12)$$

Two noteworthy implications follow from (11)–(12). First, unless  $k = 1/2$  and  $w_m = w_f = w$ , the quantitative impact of a given change in  $t^M - t^S$  on  $\theta_f^*$  and  $\theta_m^*$  depends on whether it is due to a change in  $t^M$  or in  $t^S$  (compare (8)–(9) with (11)–(12)). Second, the equilibrium effects can be qualitatively different in the two cases, as the following example illustrates.

**Example 1.** Let  $w_m = w_f = w$ , and suppose  $\theta_m$  and  $\theta_f$  are exponentially distributed with parameter  $\eta$ . It is easy to show that in this case  $\mu'_f \mu'_m - \mu_m \mu''_m = 0$  and

$\mu'_f \mu'_m - \mu_f \mu''_m = 0$ . Hence, the numerators of (11) and (12) are equal to  $w > 0$ , and thus a decrease in  $t^S$  increases both  $\theta_m^*$  and  $\theta_f^*$ . Notice, however, that an increase in  $t^M$  can lead to changes in the thresholds of opposite signs depending on the value of  $k$ . Thus, a change in  $t^M - t^S$  of a given size can affect behavior differently depending on how it is implemented.<sup>4</sup>

#### 4. Changes in differential tax treatment: welfare effects

We now address the following important question: Is it optimal to tax married and single individuals differently? An oft-heard policy recommendation calls for uniform taxation of all individuals, independently of their marital status. We show below that symmetric tax treatment of single and married people is never optimal in this framework. Under the optimal policy, the tax rate imposed on married people is *lower* than the one imposed on single individuals.<sup>5</sup>

Since the algebra becomes unwieldy in the general case, we will derive the main insights using the symmetric version of the model (i.e.,  $G_f = G_m = G$ ,  $w_f = w_m = w$ ,  $k = 1/2$ , and  $\delta_f = \delta_m$ , and hence  $U_m = U_f = U$  and  $\theta_m^* = \theta_f^* = \theta^*$ ), with  $t^S = 0$  and  $t^M$  as the choice variable. The computational experiments in Section 6, however, reveal that the same findings also emerge in the general case in which *none* of these restrictions are imposed.

We consider the problem of finding the tax rate  $t^M$  that maximizes the sum of the expected utilities of the cohort of men and women that enters the market at a given time, subject to a budget constraint that equates the present value of tax revenues generated by the cohort with the present value of expenditures associated with the cohort.

Maximizing the welfare of each ‘cohort’ of newborns subject to an intertemporal budget constraint for the cohort is a reasonable social planner’s problem to focus on for the following reasons. First, it guarantees (almost by definition) that  $t^M$  is such that the expected utility of every new entrant is maximized, which is a sensible ‘*ex ante*’ measure of welfare in this setting. Second, we shall see below that this formulation of the planner’s problem affords a tractable way to capture the ‘transition effects’ that changes in  $t^M$  impose on the marriage rate of each cohort over time, which in turn affects the revenues collected by the planner and thus the optimal choice of  $t^M$ .

Since (i) a constant fraction  $\delta$  of each population dies at every instant and is replaced by new entrants, (ii) everyone is born single, and (iii) we are considering the symmetric version of the model, it follows that the objective function of the planner is  $2\delta U$ , or simply  $U$ . Also, in order to avoid carrying the constant  $2\delta$  in our calculations (which ends up canceling out anyway), we are going to proceed as if the size of each new cohort is equal to one.

---

<sup>4</sup> The main force that drives this example is that utility is non-transferable. With transferable utility cases (ii) and (iii) of Proposition 2 cannot arise, and therefore the qualitative effects of an increase in  $t^M$  are the same as those of a decrease in  $t^S$ . The proof of this and other results when utility is transferable are available from the authors upon request.

<sup>5</sup> The analysis of this section has greatly benefited from detailed suggestions by Robert Shimer.

As we mentioned above, the constraint faced by the planner is given by a present value budget constraint for each new cohort. Let  $E$  be an exogenous level expenditure per living person in the cohort that needs to be financed. The policy instruments available to the planner are  $t^M$ , which is levied on the fraction of agents of the cohort that is alive and married at each point in time, and a tax (subsidy) rate  $t$  that is levied on all living agents of the cohort at each point in time.

Let us first calculate the present value of expenditures for each cohort. Notice that, after an interval of length  $\tau$  has elapsed, only a measure of agents  $e^{-\delta\tau}$  of the cohort remains alive. Assuming that the planner can borrow and lend at a rate  $r$ , the present value of expenditures for each cohort is given by

$$\int_0^\infty E e^{-(r+\delta)\tau} d\tau = \frac{E}{r+\delta}.$$

The revenue collected from a cohort comes from two sources, namely,  $t$  and  $t^M$ . Since  $t$  is applied to all living agents of the cohort at any given time, it follows that the present value of the revenue collected from this source is

$$\int_0^\infty t 2w e^{-(r+\delta)\tau} d\tau = \frac{t 2w}{r+\delta}.$$

To calculate the present value of the revenue generated by  $t^M$ , we need to pin down the measure of agents of the cohort that is alive and married at each time  $\tau$ . Let us denote this quantity by  $\mathcal{M}(\tau)$ . Obviously,  $\mathcal{M}(0) = 0$ , since every member of the cohort is born single. Moreover,  $\mathcal{M}(\tau)$  satisfies

$$\mathcal{M}'(\tau) = (e^{-\delta\tau} - \mathcal{M}(\tau))\gamma - 2\delta\mathcal{M}(\tau). \quad (13)$$

The solution to (13) subject to  $\mathcal{M}(0) = 0$  is given by

$$\mathcal{M}(\tau) = \frac{\gamma}{\delta + \gamma} (1 - e^{-(\gamma + \delta)\tau}) e^{-\delta\tau}.$$

This expression allows us to calculate the present value of the revenue collected from the cohort using  $t^M$  as follows:

$$\int_0^\infty t^M 2w \mathcal{M}(\tau) e^{-r\tau} d\tau = \frac{t^M 2w \gamma}{(r + \delta)(r + 2\delta + \gamma)},$$

where the last term follows by integration after substituting  $\mathcal{M}(\tau)$ .

Therefore, the balanced budget constraint on the planner's problem is

$$E = 2w \left( t + \frac{\gamma}{r + 2\delta + \gamma} t^M \right).$$

Regarding the planner's objective function, we can simplify it as follows. Notice that  $U = V(\theta^*)$  implies that

$$U = \frac{\theta^*(t^M, t) + (1 - t^M - t)w}{r + \delta}.$$

Also, since  $t$  is applied to both married and single agents, it is easy to show that  $\partial\theta^*/\partial t = 0$ .<sup>6</sup>

Using these results, the planning problem becomes:

$$\max_{t^M} \frac{\theta^*(t^M) + (1 - t^M - t(t^M))w}{r + \delta}, \quad (14)$$

where  $t(t^M) = E/2w - \gamma/(r + 2\delta + \gamma)t^M$ . We are now ready to show the main result of this section.

**Proposition 3.** *At the optimal solution of problem (14),  $t^M < 0$ .*

In other words, optimal taxation in this setting entails a marriage bonus. The key to understand this result is that, when utility is nontransferable, the matching equilibrium is inefficient. Indeed, one can show that the equilibrium threshold  $\theta^*$  is higher than the socially optimal level. To see this, notice that in the symmetric version of the model without taxes, the equilibrium threshold  $\theta^*$  solves (see (4) or (5))

$$\theta^* = -\pi\mu\mu'. \quad (15)$$

In turn, if a social planner chose the individual threshold directly, he would choose the one that solves<sup>7</sup>

$$\theta^{SP} = \frac{\mu}{2\mu'} - \frac{\pi\mu\mu'}{2}. \quad (16)$$

If we add and subtract  $\pi\mu\mu'/2$  to the right side of (16), we can rewrite it as follows:

$$\theta^{SP} = -\pi\mu\mu' + \frac{\mu}{2\mu}(1 + \pi\mu'^2). \quad (17)$$

Since  $(\mu/2\mu')(1 + \pi\mu'^2) < 0$ , it follows that the socially optimal threshold is lower than the equilibrium one. In other words, when utility is nontransferable some matching rents are left unexploited in equilibrium.

Now, the proof of Proposition 3 shows that the optimal tax rate  $t^M$  is the solution to  $t^M = (\mu/\mu')(1 + \pi\mu'^2)/(2w)$ , which rearranges to  $t^M w = (\mu/2\mu')(1 + \pi\mu'^2)$ . Thus, the size of the marriage tax bonus is *exactly* the one required to correct the aforementioned inefficiency.

<sup>6</sup> Incidentally, this shows that existence of equilibrium in the symmetric model trivially extends to the case with a balanced budget constraint. In the general case, for any given  $t$ , there exists (by Proposition 1) a unique pair  $(\theta_m^*(t), \theta_f^*(t))$ , which is continuous in  $t$ . Hence,  $\beta\mu'_f\mu'_m$  is continuous in  $t$ , and one can show that this implies that there is a  $t$  that solves the balanced budget constraint, thereby showing that an equilibrium exists.

<sup>7</sup> To derive this expression, notice that the proof of Proposition 1 shows that an agent's expected discounted utility of any given threshold  $\theta^*$  can be written as follows:

$$\frac{w + \pi\mu'^2 w - \pi\mu' \int_{\theta^*}^{\bar{\theta}} \theta dG(\theta)}{(r + \delta)(1 + \pi\mu'^2)} = \frac{w}{(r + \delta)} - \frac{\pi\mu' \int_{\theta^*}^{\bar{\theta}} \theta dG(\theta)}{(r + \delta)(1 + \pi\mu'^2)}.$$

The social planner will choose the threshold  $\theta^*$  that maximizes this expression. Straightforward manipulation of the first-order condition of this problem yields (16). Notice that we are assuming that the social planner cannot observe the actual realizations of the match specific components; all he does is to set a threshold  $\theta^*$ .

## 5. Endogenous match dissolutions

The model analyzed thus far does not allow for endogenous match dissolutions. We now incorporate divorce into the analysis and show that all the results continue to hold in this case as well.

The possibility of endogenous match dissolution is modeled as follows. According to a Poisson process with parameter  $\psi$ , both spouses draw new match-specific components of his or her current mate (i.e., the husband draws a new  $\theta_m$  and the wife a new  $\theta_f$ ) that are independent of the previous draws, and then they decide whether to continue with the match or get divorced and go back to the pool of singles.<sup>8</sup> Under this assumption, the optimal strategy for an agent is still summarized by a single threshold  $\theta_i^*$ ,  $i = m, f$ , i.e., an agent gets married if the match specific component is above the threshold and then gets divorced if the new observation falls below it.

We show in Appendix A that a matching equilibrium is a pair of thresholds  $\theta_m^*$  and  $\theta_f^*$  that solves the following system of equations:

$$\begin{aligned}\theta_m^* + k(1 - t^M)(w_m + w_f) - (1 - t^S)w_m &= -\phi \mu'_f(\theta_f^*) \mu_m(\theta_m^*), \\ \theta_f^* + (1 - k)(1 - t^M)(w_m + w_f) - (1 - t^S)w_f &= -\phi \mu'_m(\theta_m^*) \mu_f(\theta_f^*),\end{aligned}$$

where  $\phi = (\beta - \psi)/(r + \delta_f + \delta_m + \psi)$ .

The equilibrium number of married people in each population is calculated as follows: the flow into the pool of married agents is  $\beta \mu'_f(\theta_f^*) \mu'_m(\theta_m^*)(1 - M)$  while the flow out of that pool is  $(\delta_f + \delta_m + \psi(1 - \mu'_f(\theta_f^*) \mu'_m(\theta_m^*)))M$ . Therefore,

$$M^* = \frac{\beta \mu'_m \mu'_f}{(\beta - \psi) \mu'_m \mu'_f + \delta_f + \delta_m + \psi},$$

and the divorce flow is given by

$$D^* = \psi(1 - \mu'_f(\theta_f^*) \mu'_m(\theta_m^*))M^*.$$

Intuitively, an increase in  $t^M$  has the following effects on  $M^*$ . First, it decreases the income gains from marriage and prompts agents to increase the thresholds  $\theta_i^*$ ,  $i = f, m$ , thereby leading to a lower probability of accepting a match *and* to a higher probability of separation. Second, the two-sided search effect kicks in and mitigates the initial impact of an increase in  $t^M$ . If it does not offset it completely, then the net effect is a *decrease* in the equilibrium number of marriages and their duration.

Regarding the impact of an increase in  $t^M$  on the divorce flow  $D^*$ , the result is ambiguous, for the probability of divorce in general increases while the number of married people decreases.

The following result extends Propositions 1 and 2 to the case with divorce:

---

<sup>8</sup> This differs from Burdett and Wright (1998), who assume that there are *two* independent Poisson processes, one for each spouse, according to which spouses obtain new draws of their match-specific components. All the results extend to this slightly more general model, but our formulation simplifies the proofs.

**Proposition 4.** (1) Suppose that (a)  $\mu_m$  and  $\mu_f$  are log-concave functions, (b)  $\beta > \psi$ , and

$$(c) \quad E[\theta_m] > \frac{k(1-t^M)(w_f + w_m) - (1-t^S)w_m}{\phi} \quad \text{and}$$

$$E[\theta_f] > \frac{(1-k)(1-t^M)(w_f + w_m) - (1-t^S)w_f}{\phi}.$$

Then, there is a unique matching equilibrium with either  $\theta_f^* > 0$  or  $\theta_m^* > 0$  or both.

(2) If  $t^M$  increases, then either (i) both  $\theta_m^*$  and  $\theta_f^*$  increase, or (ii)  $\theta_m^*$  increases and  $\theta_f^*$  decreases, or (iii)  $\theta_m^*$  decreases and  $\theta_f^*$  increases. Case (ii) occurs if  $k$  is sufficiently close to one, while (iii) arises if  $k$  is sufficiently close to zero.

Consider now the optimal degree of differential tax treatment with divorce. For analytical tractability, we will focus again on the symmetric version of the model with  $t^S = 0$ . As before, the social planner's problem amounts to finding the tax rates  $t^M$  and  $t$  that maximize  $U$ , subject to the appropriate balanced budget constraint.

Following the same steps as in the case without divorce, it is straightforward to show that the balanced budget constraint is now given by

$$E = 2w \left( t + \frac{\gamma}{r + 2\delta + \gamma + \psi(1 - \mu'^2)} t^M \right).$$

Regarding the planner's objective function,  $U = V(\theta^*)$  and the fact that  $\partial\theta^*/\partial t = 0$  allow us to write it as follows:

$$U = \frac{\theta^*(t^M) + (1 - t^M - t)w - \frac{\psi\mu'\mu}{r+2\delta+\psi}}{r + \delta}.$$

Thus, the planning problem is given by

$$\max_{t^M} \frac{\theta^*(t^M) + (1 - t^M - t(t^M))w - \frac{\psi\mu'\mu}{r+2\delta+\psi}}{r + \delta}, \quad (18)$$

where  $t(t^M) = E/2w - \gamma t^M/(r + 2\delta + \gamma + \psi(1 - \mu'^2))$ .

We are now ready to extend Proposition 3 to the case with divorce.

**Proposition 5.** At the optimal solution of problem (18),  $t^M < 0$ .

As in Section 4, this result implies that symmetry in the tax treatment of individuals is not optimal. The intuition underlying the optimality of a marriage tax bonus is analogous to the one given after Proposition 3 and will not be repeated.

## 6. Quantitative results

We now turn to the computation of the model with and without endogenous match dissolutions. We illustrate the quantitative implications of the model regarding the number

of marriages, divorces, and the behavior towards marriage associated with changes in taxes. Needless to say, it is important to keep in mind that the environment we analyze abstracts from several potentially important features of the marriage market.

We use actual US data to restrict our choice of the parameters of the model whenever possible. Regarding the match-specific components  $\theta_m$  and  $\theta_f$ , in this section we simply assume a particular distribution and then conduct a sensitivity analysis with respect to the parameters that characterize it. We will come back to this issue in Section 7.

### 6.1. Equilibrium effects of changes in $t^M$

We present numerical results when  $t^M$  is increased, given a fixed value of  $t^S$ .

#### *The benchmark model*

Consider first the benchmark case without endogenous match dissolutions. We set the model period equal to a year and  $t^S = 0.20$ . We then choose the other parameters as follows. Regarding mortality rates, we assume that males and females enter marriage and labor markets at age 20. Thus, we set mortality rates based on life-expectancy conditional upon being alive at such age. US data indicates a life expectancy of 55.2 years for males and 60.3 for females in 2000 (Statistical Abstract of the United States, US Census Bureau, 2002, Table 92). This yields  $\delta_m = 0.0181$  and  $\delta_f = 0.0166$ . The annual value for the rate of time preference  $r$  is set equal to 0.04. We normalize  $w_f = 1.0$  and choose  $w_m$  so as to match the ratio of mean earnings of males to mean earnings of females. The average of this ratio is about 0.67 for the period 1990–1998 (US Census Bureau, Historical Income Tables, Table P-39. Individuals in the sample are “full-time, year-round workers”). Consequently, we set  $w_m = 1.4925$ .

We assume that match specific components are exponentially distributed with parameter  $\eta_m = \eta_f = \eta$ , and hence  $\mu_f$  and  $\mu_m$  are log-concave. A nice implication of this assumption is that it makes all the results concerning the equilibrium effects on the number of marriages associated with changes in  $t^M$  revenue neutral. The logic is the following: under the exponential assumption,  $\mu''_f \mu_m = \mu''_f \mu_m$  and  $\mu''_f \mu'_m = \mu''_f \mu'_m$ , which in turn imply that  $\partial \theta_m^*/\partial t = -\partial \theta_f^*/\partial t$  and hence  $\partial M^*/\partial t = 0$ . Thus, we can always adjust  $t$  to balance the budget without any effects on  $M^*$ .

The parameter  $\eta$  is chosen so that the mean of match specific components ( $1/\eta$ ) is in line with the level of income of a married couple. With no endogenous match dissolutions, we set  $1/\eta$  equal to the fraction of income that each spouse obtains under equal division ( $k = 0.5$ ), namely  $(w_f + w_m)/2$ .

To facilitate the comparison with the case with divorce and the mapping of the model to the data, it will prove convenient to assume in the benchmark case that there is an additional Poisson process with parameter  $\rho$  that exogenously destroys matches; i.e.,  $\lambda_i = \delta_i + \rho$ .

The parameter  $\rho$  is set so as to reproduce the overall expected duration of a marriage  $((\delta_m + \delta_f + \rho)^{-1})$  found in the data. For this statistic, Schoen and Standish (2001) report a value of 25.7 years. This dictates, given  $\delta_m$  and  $\delta_f$ , a value for  $\rho$  of 0.00467.

The remaining parameter of the model, the contact rate  $\beta$ , is set so that under equal division ( $k = 0.5$ ) and with  $t^M = t^S$ , the equilibrium fraction of married individuals  $M^*$  approximates observed value for 1990–2000 (60.8%).<sup>9</sup> The resulting value is  $\beta = 0.2845$ .

Table 1 presents the results regarding the stock of marriages, threshold values and acceptance rates for males and females ( $(1 - G_i(\theta_i^*))$ ,  $i = f, m$ ). Two features are worth pointing out. First, notice the relative insensitivity of the equilibrium fraction of married individuals for different values of  $t^M$ . For example, when  $k = 0.5$  a change of eight percentage points in  $t^M$  (from 0.16 to 0.24), which changes  $t^M - t^S$  from  $-0.04$  to  $0.04$ , generates a reduction of about one and a half percentage points in  $M^*$ . If we measure the impact of the marriage tax on marriage behavior using the elasticity of the stock of marriages with respect to  $t^M - t^S$ , its value is merely  $-0.0123$ .<sup>10</sup> Second,  $M^*$  always decreases with an increase in  $t^M$ , and for  $k = 0.30$  and  $k = 0.70$  one of the thresholds can actually fall when  $t^M$  increases, illustrating the results of Proposition 2. For  $k$  sufficiently close to one ( $k = 0.7$ ), the threshold for males increases and the threshold for females decreases, while the opposite happens when  $k$  is sufficiently close to zero ( $k = 0.30$ ). Notice that  $M$  is not affected by changes in  $k$ . This is also an implication of the exponential distribution, since  $\mu_f''\mu_m = \mu_f''\mu_m$  and  $\mu_f''\mu'_m = \mu_f''\mu'_m$  imply that  $\partial\theta_m^*/\partial k = -\partial\theta_f^*/\partial k$  and  $\partial M/\partial k = 0$ .

### *Endogenous match dissolutions*

Assuming  $k = 0.5$ ,  $t^M = t^S = 0.2$ , and exponentially distributed match specific components, we now choose  $\psi$  and  $\beta$  to match the same two US statistics we targeted before, namely, the steady state number of marriages and the expected duration of a marriage,  $(\delta_m + \delta_f + \psi(1 - \mu_m(\theta_m^*)\mu_f(\theta_f^*)))^{-1}$ . Since divorce is now endogenous, we need to search across equilibria over these two parameters simultaneously. Table 2 presents the results for different values of  $t^M$  and  $k$ .

Notice that now the expected duration of a marriage is affected (negatively) by the changes in the marriage tax penalty. The magnitude of this effect, however, is relatively small. We also note that  $M^*$  becomes slightly more sensitive to changes in  $t^M - t^S$  relative to the benchmark case, albeit the magnitudes are still small. For instance, when  $k = 0.5$ , an increase of eight percentage points in the marriage tax (from  $-0.04$  to  $0.04$ ) reduces the number of marriages by about one and a half percentage points. Measuring the changes via elasticities, the elasticity goes up in absolute value, from  $-0.0123$  in the benchmark case to about  $-0.0131$  under endogenous matching dissolution. Thus, the relative insensitivity of  $M^*$  with respect to changes in the marriage tax found previously is still present in the model with divorce.

We also note that the thresholds can move in opposite directions. Moreover, a noteworthy finding that emerges from Table 2 is that the number of divorces actually falls with  $t^M$ , albeit slightly. In other words, a decrease in the marriage tax penalty can actually *increase* the number of divorces across steady states.

<sup>9</sup> Individuals considered are 18 years old or over. Source: Statistical Abstract of the US (US Census Bureau, 2002, Table 46).

<sup>10</sup> The elasticities reported in Table 1 are calculated for ‘large’ changes (from  $-0.04$  to  $0.04$ ) in the marriage tax. If they were calculated for ‘small’ changes (for instance, in the neighborhood of  $t^M = t^S$ ), the resulting elasticities would be even smaller in absolute value.

Table 1

Stock of marriages, individual thresholds and acceptance rates ( $t^S = 0.20$ )

Tax on married couples ( $t^M$ )	$k = 0.30$	$k = 0.50$	$k = 0.70$
16%			
Fraction married	0.6154	0.6154	0.6154
$\theta_f^*$	0.3235	0.7424	1.1613
$\theta_m^*$	1.5554	1.1363	0.7175
Acceptance rate (fem.)	0.7714	0.5512	0.3938
Acceptance rate (male)	0.2871	0.4018	0.5623
18%			
Fraction married	0.6117	0.6117	0.6117
$\theta_f^*$	0.3431	0.7617	1.1609
$\theta_m^*$	1.5551	1.1561	0.7372
Acceptance rate (fem.)	0.7774	0.5470	0.3939
Acceptance rate (male)	0.2871	0.3986	0.5535
20%			
Fraction married	0.6080	0.6080	0.6080
$\theta_f^*$	0.3629	0.7519	1.1611
$\theta_m^*$	1.5549	1.1463	0.7569
Acceptance rate (fem.)	0.7474	0.5470	0.3939
Acceptance rate (male)	0.2871	0.3986	0.5448
22%			
Fraction married	0.6042	0.6042	0.6042
$\theta_f^*$	0.3827	0.7716	1.1607
$\theta_m^*$	1.5548	1.1659	0.7767
Acceptance rate (fem.)	0.7356	0.5384	0.3940
Acceptance rate (male)	0.2872	0.3924	0.5362
24%			
Fraction married	0.6004	0.6004	0.6004
$\theta_f^*$	0.4027	0.7815	1.1607
$\theta_m^*$	1.5548	1.1759	0.7966
Acceptance rate (fem.)	0.7239	0.5341	0.3940
Acceptance rate (male)	0.2872	0.3892	0.5277
Elasticity ( $M^*, t^M - t^S$ )	-0.0123	-0.0123	-0.0123

*Relative importance of income and match-specific components*

We now evaluate the sensitivity of the results with respect to the assumption that the mean of match specific components is equal to half of the couple's income. This exercise is important, for it is natural to expect that individuals will become more sensitive to changes in the marriage tax if the income change associated with marriage becomes relatively more important. In principle, this could alter the relatively small effect of an increase in  $t^M$  on the number of marriages found previously.

We focus only on the case where  $k = 0.5$  and compute the equilibrium of the model under the assumption that the mean of the distribution of the match specific components is equal to a fraction  $a$  of the income that each spouse obtains when married, given by

Table 2

Endogenous divorce: number of marriages and divorces, individual thresholds and marriage duration ( $t^S = 0.20$ )

Tax on married couples ( $t^M$ )	$k = 0.30$	$k = 0.50$	$k = 0.70$
16%			
Fraction married	0.6158	0.6158	0.6158
Divorce flow	0.00284	0.00284	0.00284
Marriage duration	25.73	25.73	25.73
$\theta_f^*$	0.2850	0.7039	1.1226
$\theta_m^*$	1.5169	1.0978	0.6791
18%			
Fraction married	0.6119	0.6119	0.6119
Divorce flow	0.00284	0.00284	0.00284
Marriage duration	25.72	25.72	25.72
$\theta_f^*$	0.3049	0.7138	1.1225
$\theta_m^*$	1.5168	1.1078	0.6990
20%			
Fraction married	0.6080	0.6080	0.6080
Divorce flow	0.00283	0.00283	0.00283
Marriage duration	25.70	25.70	25.70
$\theta_f^*$	0.3249	0.7238	1.1226
$\theta_m^*$	1.5169	1.1178	0.7190
22%			
Fraction married	0.6039	0.6039	0.6039
Divorce flow	0.00283	0.00283	0.00283
Marriage duration	25.69	25.69	25.69
$\theta_f^*$	0.3450	0.7339	1.1227
$\theta_m^*$	1.5170	1.1279	0.7391
24%			
Fraction married	0.5998	0.5998	0.5998
Divorce flow	0.00282	0.00282	0.00282
Marriage duration	25.67	25.67	25.67
$\theta_f^*$	0.3651	0.7441	1.1229
$\theta_m^*$	1.5170	1.1381	0.7593
Elasticity ( $M^*, t^M - t^S$ )	-0.0131	-0.0131	-0.0131

$(w_m + w_f)/2$ . Table 3 reports the corresponding changes in  $M^*$  and the elasticity of  $M^*$  with respect to  $t^M - t^S$  for  $a \in \{0.5, 1.0, 1.5\}$ .<sup>11</sup>

Intuitively, as  $a$  decreases and income considerations become relatively more important, the elasticity increases in absolute value. In other words, the behavior of individuals towards marriage becomes more sensitive to changes in the marriage tax when the match-specific component is less important. Quantitatively, the changes in the impact of the

<sup>11</sup> The contact rate implicit in the calculations in Table 3 is the same as in Tables 1 and 2. The elasticities do not change significantly if the contact rate is adjusted to match, for each level of  $a$ , the target number of marriages under  $k = 0.5$  and  $t^S = t^M = 0.20$ .

Table 3  
Monetary vs. match-specific components ( $k = 0.5$ )

	$a = 0.5$	$a = 1.0$	$a = 1.5$
<i>Exogenous divorce</i>			
Change in $M^*$	−0.030	−0.015	−0.010
Elasticity	−0.0247	−0.0123	−0.0082
$(M^*, t^M - t^S)$			
<i>Endogenous divorce</i>			
Change in $M^*$	−0.032	−0.016	−0.011
Elasticity	−0.0263	−0.0131	−0.0088
$(M^*, t^M - t^S)$			

marriage tax penalty are still small. A reduction of 50% in the importance of match specific components (from  $a = 1.0$  to  $a = 0.5$ ) yields an increase in the change in the number of marriages (associated to the change of the marriage tax penalty from −0.04 to 0.04) from 0.015 to 0.03 with exogenous match dissolution. With divorce, the increase in the change in the number of marriages goes from 0.016 to 0.032.

## 6.2. Optimal differential tax treatment

We now turn to find the values of the tax rates that maximize the sum of the expected discounted welfare of newborn individuals. We proceed as follows. We again set  $t^S = 0.20$  and allow the planner to find the optimal  $t^M$  and  $t$  subject to the balanced budget constraint. The values of the remaining parameters are set as before.

The main finding is that the optimal gap between taxes on married and single people is quite large. We find that the tax rate on married couples in the benchmark case is *negative* and approximately −109.6%. The corresponding tax on all individuals is about 81.2%. If we restrict attention to taxes on single individuals that satisfy  $t + t^S \leq 1$ , then  $t$  and  $t^M$  are, approximately, 80% and −108.0%. This effectively implies that taxes on singles are equal 100% of their income, and that married couples face a negative tax, or government transfer, equal to −28.0%. Very similar results hold in our version of the model with endogenous divorce, and also when the budget is balanced using  $t^S$  instead of  $t$ .

The intuition underlying the large magnitude of the optimal marriage bonus found is as follows. In Section 4, we showed that the optimal marriage bonus can be understood in terms of the planner's desire to increase the number of marriages to its efficient level. Under the optimal tax policy, the number of marriages ( $M^*$ ) is 0.771, which is a much larger figure than our calibration target ( $M^* = 0.608$ ). Given the relative insensitivity of the number of marriages to changes in differential tax treatment, it is not surprising that a large gap between  $t^M$  and  $t^S$  is needed to accomplish such a dramatic increase in  $M^*$ .

It is important to keep in mind that these results are derived in a model in which differential tax treatment *only* affects marriage decisions. To be sure, if taxes also affected other decisions such as labor supply or capital accumulation, the resulting optimal degree of differential tax treatment will probably be smaller.

As an illustration, consider the symmetric case analyzed in Section 4. With  $t^S = 0$ , one can verify than in the exponential case the optimal tax rate is the unique solution to:

$$t^M = -\frac{E[\theta]}{2w} \left( 1 + \pi e^{-\eta\theta^*(t^M)} \right).$$

In particular, notice that if  $E[\theta] \geq w$ , the tax on marriage couples is lower than  $-50\%$ !

To shed additional light on these numerical results, it is important to mention that, in terms of welfare, both populations fare differently under the optimal policy. Females gain while males are worse off in the social optimum relative to the initial situation. If, hypothetically, the planner's objective were to maximize only the welfare of males, the results would be very different than the ones reported above. Indeed, the optimal tax rates in this case would be  $t^M = -14.2\%$  and  $t = 17.0\%$ . More generally, the quantitative analysis of the model reveals that large changes in welfare of opposite sign take place under the planning solution when  $w_m \neq w_f$ . As  $w_m$  grows relative to  $w_f$ , males dislike being heavily taxed with  $t$  and prefer higher values of  $t^M$  instead.

## 7. The cohabitation margin

So far we have not allowed for cohabitation as an alternative to marriage. This might be an important consideration for the questions under analysis. The presence of cohabitation provides an additional margin that adjusts with changes in the tax structure. For instance, if  $t^M$  increases, then couples who would otherwise marry could choose to cohabit instead, thereby making the number of marriages more sensitive to changes in the tax treatment of married and single individuals.

To the best of our knowledge, there are no theoretical models in which marriage and cohabitation coexist. To be sure, the inclusion of cohabitation in a marriage model is a challenging analytical task. In addition, a model of cohabitation and marriage must leave room to account for a number of critical observations. Consider the following facts about cohabitation in the US. First, only a small fraction of the US adult population, approximately 4%, cohabits at a given point in time (Bumpass and Sweet, 1989, Table 1). In contrast, the fraction of married people in the adult population is more than twelve times larger. Second, US evidence indicates that, in terms of duration, cohabitation is fundamentally different from marriage: the mean duration of a cohabiting relationship is only about 3 years (Bumpass and Sweet, 1989; Bumpass et al., 1991; Bramlett and Mosher, 2002). Third, the evidence also indicates that cohabitation is closely linked to marriage, for a large fraction of cohabitation experiences transits into marriage relationships.<sup>12</sup> As a result, a large fraction of marriages in recent cohorts are preceded by a cohabitation experience.<sup>13</sup> Finally, the evidence suggests that marriages preceded by cohabitation need not be more stable.<sup>14</sup>

<sup>12</sup> Bumpass and Sweet (1989, Table 4), calculate that the probability that a cohabitation relationship makes a transition to marriage is approximately 0.25 after 1 year, 0.56 after 5 years and 0.59 after 10 years.

<sup>13</sup> For instance, Bumpass and Lu (2000) calculate that, for the union cohort 1990–1994, about 40 percent of women aged 19–44 cohabited with their first husband.

<sup>14</sup> See Cherlin (1992, Chapter 1), and the references therein for a discussion.

While a complete analysis of a model of marriage and cohabitation is beyond the scope of this paper, we present below a stylized framework that captures some aspects of the problem that are important for the questions at hand. Such a model permits marriage and cohabitation to coexist in equilibrium, and it allows individuals to experience a number of transitions in their marital status. Over time, they can transit from being single to cohabiting or to being married, and from cohabiting to being married.

### *A model of marriage and cohabitation*

We now consider an extension of the previous model to allow for the possibility of cohabitation. When two singles meet, after each observes the realization of the match-specific component, they simultaneously announce ‘single,’ ‘cohabitation,’ or ‘marriage.’ If both announce ‘cohabitation’ (‘marriage’), then they form a match and cohabit (get married). In any other event they remain single and continue searching for a partner.

Under cohabitation, an agent enjoys his or her match-specific component and pays taxes as a single person (i.e., according to  $t^S$ ). There are also two independent shocks that can alter his or her cohabitation status. First, according to a Poisson process with arrival rate  $\lambda_i$ ,  $i = m, f$ , cohabitation breaks down and an agent of type  $i$  becomes single again. Second, according to a Poisson process with arrival rate  $\psi$ , both partners simultaneously draw new realizations of the match-specific components, and then announce ‘single,’ ‘cohabitation,’ or ‘marriage.’ If both announce ‘cohabitation,’ then they continue cohabiting but with the new match-specific components. If both announce ‘marriage,’ cohabitation ends and the couple gets married and enjoys the new match-specific components. In any other event cohabitation ends and they become single again.

When married, an agent enjoys his or her match-specific component and pays taxes as a married person (i.e., according to  $t^M$ ). As in Section 2, according to a Poisson process with arrival rate  $\lambda_i$ ,  $i = m, f$ , the marriage ends and an agent of type  $i$  becomes single again.

Before we turn to a more formal description of this extension, the following comments about the trade-offs present in the model are in order.

Notice that, compared to marriage, cohabitation allows an agent to continue paying taxes as a single person while enjoying the match-specific component. If  $t^M > t^S$ , this is obviously advantageous. Moreover, it offers an agent the valuable possibility of transiting into marriage or continue cohabiting but with a higher match-specific component. The downside is that, unlike marriage, cohabitation is subject to a higher rate of match destruction, for it can be dissolved exogenously as well as endogenously. In other words, cohabitation is a more *unstable* arrangement than marriage.

Similarly, compared to being single, cohabitation allows an agent to enjoy the match-specific component and also the option value of transiting into marriage or continue cohabiting but with higher match-specific component. But being single is not necessarily dominated by cohabitation if  $\beta > \psi$  or if the share of the total income the agent obtains (which depends on  $k$ ) is small.

Consider the decision problem faced by a male in this environment. Let  $U_m$  be the expected discounted utility of being single, and let  $V_m^C(\theta_m)$ , and  $V_m^M(\theta_m)$  be the expected discounted utility of cohabitation and marriage with match-specific component  $\theta_m$ . Then,

$$(r + \delta_m)U_m = (1 - t^S)w_m + \beta E[\max\{0, \xi_m^C(V_m^C(\theta_m) - U_m), \xi_m^M(V_m^M(\theta_m) - U_m)\}], \quad (19)$$

$$\begin{aligned} (r + \delta_m)V_m^C(\theta_m) &= k(1 - t^S)(w_m + w_f) + \theta_m + (\lambda_m + \psi)(U_m - V_m^C(\theta_m)) \\ &\quad + \psi E[\max\{0, \xi_m^C(V_m^C(\theta_m) - U_m), \xi_m^M(V_m^M(\theta_m) - U_m)\}], \end{aligned} \quad (20)$$

$$(r + \delta_m)V_m^M(\theta_m) = k(1 - t^M)(w_m + w_f) + \theta_m + \lambda_m(U_m - V_m^M(\theta_m)), \quad (21)$$

where  $\xi_m^C$  and  $\xi_m^M$  are the probabilities that a woman announces ‘cohabitation’ and ‘marriage.’ Women solve a similar problem and hence there are analogous expressions for  $U_f$ ,  $V_f^C(\theta_f)$ , and  $V_f^M(\theta_f)$ .

Obviously, there are always trivial equilibria in which cohabitation does not emerge. For instance, if men conjecture that women will never announce ‘cohabitation,’ then it is a best response for women not to announce ‘cohabitation’ either, and vice versa. To avoid this triviality, we will search for a matching equilibrium characterized by a four-tuple  $(\theta_m^C, \theta_m^M, \theta_f^C, \theta_f^M)$ , with  $0 \leq \theta_m^C < \theta_m^M < \bar{\theta}$  and  $0 \leq \theta_f^C < \theta_f^M < \bar{\theta}$ . That is, the optimal strategy for an agent of type  $i$ ,  $i = m, f$ , is to announce ‘single’ if  $\theta_i < \theta_i^C$ , ‘cohabitation’ if  $\theta_i^C \leq \theta_i < \theta_i^M$ , and ‘marriage’ if  $\theta_i \geq \theta_i^M$ . Moreover, these thresholds satisfy  $V_i^C(\theta_i^C) = U_i$  and  $\xi_i^C(V_i^C(\theta_i^M) - U_i) = \xi_i^M(V_i^M(\theta_i^M) - U_i)$ ,  $i = m, f$ .

In Appendix A we show that a matching equilibrium with these features solves the following system of equations:

$$\begin{aligned} \theta_i^C &= \frac{(1 - \psi\xi_i^M\sigma_i^M\mu'_i(\theta_i^M))(1 - t^S)w_i - (1 - \beta\xi_i^M\sigma_i^M\mu'_i(\theta_i^M))w_i(k)(1 - t^S)}{1 + (\beta - \psi)\xi_i^C\sigma_i^C(\mu'_i(\theta_i^M) - \mu'_i(\theta_i^C)) - \beta\xi_i^M\sigma_i^M\mu'_i(\theta_i^M)} \\ &\quad + \frac{(\beta - \psi)(\xi_i^C\sigma_i^C\int_{\theta_i^C}^{\theta_i^M}\theta_i dG_i(\theta_i) + \xi_i^M\sigma_i^M\int_{\theta_i^M}^{\bar{\theta}_i}\theta_i dG_i(\theta_i) - \xi_i^M\sigma_i^M\mu'_i(\theta_i^M)w_i(k)(1 - t^S))}{1 + (\beta - \psi)\xi_i^C\sigma_i^C(\mu'_i(\theta_i^M) - \mu'_i(\theta_i^C)) - \beta\xi_i^M\sigma_i^M\mu'_i(\theta_i^M)}, \\ \theta_i^M &= \theta_i^C + \frac{\xi_i^M\sigma_i^M(\psi\xi_i^M\sigma_i^M(\mu'_i(\theta_i^M)\theta_i^C + \int_{\theta_i^C}^{\bar{\theta}_i}\theta_i dG_i(\theta_i)) + w_i(k)(t^M - t^S) + \psi\xi_i^C\sigma_i^C\int_{\theta_i^C}^{\theta_i^M}\theta_i dG_i(\theta_i))}{(1 - \psi\xi_i^M\sigma_i^M\mu'_i(\theta_i^M))(\xi_i^M\sigma_i^M - \xi_i^C\sigma_i^C)}, \end{aligned}$$

where  $i = m, f$ ;  $\sigma_i^C = 1/(r + \delta_i + \lambda_i + \psi)$ ;  $\sigma_i^M = 1/(r + \delta_i + \lambda_i)$ ;  $w_m(k) = k(w_m + w_f)$ , and  $w_f(k) = (1 - k)(w_m + w_f)$ ;  $\xi_m^C = \mu'_f(\theta_f^M) - \mu'_f(\theta_f^C)$ ;  $\xi_f^C = \mu'_m(\theta_m^M) - \mu'_m(\theta_m^C)$ ;  $\xi_m^M = -\mu'_f(\theta_f^M)$ ,  $\xi_f^M = -\mu'_m(\theta_m^M)$ ,  $\lambda_m = \delta_f$ ; and  $\lambda_f = \delta_m$ .

If a matching equilibrium with these characteristics exists, then the steady-state measures of agents of each population that are single ( $S$ ), cohabiting ( $C$ ), and married ( $M$ ) can be calculated as follows. The number of people of each population that flows into cohabitation is  $\beta\xi_m^C\xi_f^C S$ , while the number that flows out is  $(\delta_m + \delta_f + \psi(1 - \xi_m^C\xi_f^C))C$ . Similarly, the flow into marriage is  $\beta\xi_m^M\xi_f^M S + \psi\xi_m^M\xi_f^M C$ , while the number that flows out is  $(\delta_m + \delta_f)M$ . Thus, in equilibrium we have

$$\beta\xi_m^C\xi_f^C S = (\delta_m + \delta_f + \psi(1 - \xi_m^C\xi_f^C))C,$$

$$\beta\xi_m^M\xi_f^M S + \psi\xi_m^M\xi_f^M C = (\delta_m + \delta_f)M,$$

$$S + C + M = 1.$$

Solving for  $S$ ,  $C$ , and  $M$ , we obtain:

$$S^* = \frac{(\delta_m + \delta_f)(\delta_m + \delta_f + \psi(1 - \xi_m^C \xi_f^C))}{\Delta}, \quad (22)$$

$$C^* = \frac{\beta \xi_m^C \xi_f^C (\delta_m + \delta_f)}{\Delta}, \quad (23)$$

$$M^* = \frac{\beta \xi_m^M \xi_f^M (\beta \xi_m^C \xi_f^C + \delta_m + \delta_f + \psi(1 - \xi_m^C \xi_f^C)) - (\beta - \psi) \xi_m^M \xi_f^M \beta \xi_m^C \xi_f^C}{\Delta}, \quad (24)$$

where  $\Delta$  is equal to the sum of the numerators of (22), (23), and (24).

A complete analytical equilibrium characterization is beyond reach, so we instead proceed to report some of the quantitative implications of the model.

### *Quantitative implications*

In order to match the model with the data, it will be convenient to assume that there are two additional independent Poisson processes, one with parameter  $\rho^C$  and the other with parameter  $\rho^M$ , which exogenously destroy cohabitation relationships and marriages.

We consider three cases, which are defined by the relative magnitudes of  $\psi$  and  $\beta$ . These cases are: (i)  $\psi = 0.90 \times \beta$ ; (ii)  $\psi = \beta$ ; and (iii)  $\psi = 1.10 \times \beta$ . In case (ii), it is easy to show that, since  $w_m > w_f$ ,  $\theta_m^C > 0$  while  $\theta_f^C = 0$ ; the quantitative analysis reveals that the same is true for case (i).

Since the arrival rates  $\psi$  and  $\beta$  are related in this way, it follows that there are three parameters to pin down, namely, the parameter  $\eta$  of the exponential distribution of  $\theta_m$  and  $\theta_f$ , the exogenous cohabitation destruction rate  $\rho^C$ , and the contact rate  $\beta$ . We search for values of these parameters that reproduce, in steady-state, three statistics that take into account the cohabitation phenomenon:

- (1) the fraction of agents of each population that cohabits (4%);<sup>15</sup>
- (2) the mean duration of a cohabitation relationship (3.1 years);<sup>16</sup> and (3) the fraction of people from each population that is married (60.8%).<sup>17</sup>

Table 4 displays the parameters that reproduce the observations when  $t^S = t^M = 0.2$  in the equal division case ( $k = 0.5$ ). The other parameters of the model, including  $\rho^M$ , are set equal to the same values used before.

<sup>15</sup> Source: Bumpass and Sweet (1989, Table 1). More recent data compiled by Fields and Casper (2001) shows a similar fraction of adults cohabiting at a point in time: 3.7%. As these authors recognize, the true fraction might be higher due to reporting problems. We thus target the aforementioned 4% figure.

<sup>16</sup> Our calculations come from data in Bumpass and Sweet (1989, Table 4).

<sup>17</sup> Ideally, one would like to consider four observations to pin down four parameters  $(\beta, \psi, \rho^C, \eta)$ . This proved to be difficult, so we chose to reduce the parameter space by relating the magnitudes of  $\beta$  and  $\psi$ . The natural observation to add would be a transition probability from cohabitation to marriage. In the data, transition probabilities decline strongly with cohabitation duration; the fraction of a cohort of agents who cohabit that transits to marriage is 0.25, 0.41, 0.48, 0.52, 0.56 and 0.59 in 1, 2, 3, 4, 5 and 10 years, respectively (Bumpass and Sweet, 1989, Table 4). Our simple model, unfortunately, implies a constant transition probability. For example, under Case 2 ( $\beta = \gamma$ ), the model implies fractions of 0.468 and 0.612, in 10 and 15 years respectively.

Table 4  
Cohabitation model: parameter values

Parameter	Case 1	Case 2	Case 3
Mean match specific component	1.3089	1.4204	1.5331
Match arrival rate ( $\beta$ )	0.2538	0.2335	0.2398
Match arrival rate ( $\psi$ )	0.2284	0.2335	0.2180
Cohabitation destruction rate ( $\rho^C$ )	0.1319	0.1308	0.1285

Table 5  
Cohabitation model: married and cohabiting individuals ( $t^S = 0.20$ )

Tax on married couples ( $t^M$ )	Case 1	Case 2	Case 3
16%			
Fraction married ( $M^*$ )	0.6260	0.6232	0.6202
Fraction cohabiting ( $C^*$ )	0.0342	0.0355	0.0364
$M^* + C^*$	0.6603	0.6586	0.6566
Duration( $C^*$ )	3.0703	3.0751	3.0782
18%			
Fraction married ( $M^*$ )	0.6171	0.6159	0.6143
Fraction cohabiting ( $C^*$ )	0.0371	0.0377	0.0381
$M^* + C^*$	0.6541	0.6536	0.6524
Duration( $C^*$ )	3.0850	3.0877	3.0887
20%			
Fraction married ( $M^*$ )	0.6078	0.6084	0.6080
Fraction cohabiting ( $C^*$ )	0.0400	0.0400	0.0400
$M^* + C^*$	0.6478	0.6483	0.64804
Duration( $C^*$ )	3.1000	3.1000	3.1000
22%			
Fraction married ( $M^*$ )	0.5979	0.6005	0.6016
Fraction cohabiting ( $C^*$ )	0.0432	0.0425	0.0420
$M^* + C^*$	0.6412	0.6430	0.6436
Duration( $C^*$ )	3.1168	3.1144	3.1113
24%			
Fraction married ( $M^*$ )	0.5876	0.5923	0.5948
Fraction cohabiting ( $C^*$ )	0.0467	0.0452	0.0441
$M^* + C^*$	0.6343	0.6374	0.6389
Duration( $C^*$ )	3.1341	3.1288	3.1235
Elasticity ( $M^*, t^M - t^S$ )	-0.0317	-0.0254	-0.0209

In line with our previous analysis, we consider changes in  $t^M$  from 16% to 24% for a given tax rate on singles  $t^S = 0.20$ . Table 5 shows the effects this change has on  $M^*$ ,  $C^*$ , the mean duration of cohabitation, and the total fraction of individuals in a match  $M^* + C^*$ .

Notice that, albeit the magnitude of changes in the number of marriages is still moderate, it is *larger* when the cohabitation margin is present than in the benchmark case without cohabitation. Now an increase in  $t^M$  from 0.16 to 0.24 reduces the number of marriages between 2.5 and 3.8 percentage points. In contrast, in the benchmark case with exogenous divorce the reduction is about one and a half percentage points. Measured by elasticities,

Table 6

Benchmark results with match specific components from cohabitation model

	Benchmark	Case 1	Case 2	Case 3
Change in $M^*$	-0.015	-0.014	-0.013	-0.012
Elasticity ( $M^*, t^M - t^S$ )	-0.0123	-0.0118	-0.0108	-0.0100

the tax changes lead to elasticities ranging from -0.0209 to -0.0317. The corresponding ones for the benchmark case with exogenous divorce is -0.0123.

This result is intuitive. Given a tax rate on married couples  $t^M$ , an individual who draws a match-specific component marginally above the marriage threshold accepts to enter into a marriage relationship. Consider an increase  $t^M$ . Now an individual who draws the same value as before rejects the prospective match as a marriage relationship, for the after tax income has fallen. But since cohabitation is taxed with  $t^S$ , and the cohabitation threshold is lower than the marriage threshold, this individual is willing to accept the potential partner as a cohabitor. Put differently, an increase in  $t^M$  increases the marriage threshold, and hence shifts some relationships ‘in the margin’ from marriage to cohabitation. This mechanism leads to an equilibrium reduction in the number of marriages and an increase in the number of cohabitation relationships.

Regarding the total number of individuals cohabiting, its magnitude increases with  $t^M$ . Indeed, this increase is nontrivial: the change in the number of individuals cohabiting ranges from 3.4% to 4.6% in Case 1, and from 3.6% to 4.4% in Case 3. We note that this effect compensates the reduction in the number of marriages. The net result is that the total number of individuals in a match, given by  $C^* + M^*$ , actually falls but by a magnitude that is smaller than the decrease in the number of marriages, with a reduction ranging from 2.6 to 1.8 percentage points.

We close this section with the following comment. The quantitative results of Section 6 are derived under the *assumption* that the mean value of match specific components,  $1/\eta$ , is equal to the mean income of the spouses, i.e.,  $0.5 \times (w_m + w_f) = 0.5 \times (1.4925 + 1) = 1.24625$ . Since the introduction of cohabitation allows us to *pin down* this parameter (see Table 4), the reader may wonder how the results of Section 6 would change if we recalculated them using the value of  $\eta$  found in this section. We conducted this exercise and, for completeness, we report in Table 6 a summary of the results obtained. In line with our previous sensitivity analysis, the resulting changes in the stock of marriages are smaller than in the benchmark case. This is unsurprising since, in all cases, the estimated mean of the distribution of match specific components is higher than the one assumed in the benchmark case of Section 6.

## 8. Concluding remarks

This paper analyzes positive and normative effects of differential tax treatment of married and single people in a marriage market model characterized by search frictions and nontransferable utility.

A central finding that emerges from our analysis is that changes in differential tax treatment are associated with relatively small changes in the number of marriages. As

our numerical examples illustrate, this result is robust to the consideration of endogenous divorce and cohabitation. In this regard, we note that there has been much discussion about the significant changes in marriage patterns in the US, and on the role played by tax penalties/bonuses as key driving forces behind these changes. For instance, the number of married individuals fell from nearly 72% of the adult population in 1950 to about 60% in 2000. Our model strongly suggests that changes of this magnitude *cannot* be attributed to observed changes in differential tax treatment of married and single individuals. In Chade and Ventura (2002), we also found a relatively small change in the number of marriages associated with changes in differential tax treatment, including its complete elimination. Therefore, the role that differential tax treatment has played on the changes in marriage patterns observed in the US should be viewed as being of secondary importance.

Also noteworthy is that our model of marriage and cohabitation shows that cohabitation rises rapidly as the tax rate on married couples increases. This is of interest given the rise in the importance of cohabitation in recent years. We note that this rise is contemporaneous with an increase in the number of two-earner couples and hence in the incidence of the marriage tax. A conjecture to explore is the extent to which these phenomena are related. To be sure, this requires a more complete model of cohabitation and marriage.

Another important result that emerges from the theoretical analysis is that an optimal tax program does not entail symmetric tax treatment of married and single people. We showed that the optimal tax schedule unambiguously requires a tax-induced marriage bonus. To the best of our knowledge, this is the first theoretical characterization of optimal taxes on married and single people we are aware of. At the quantitative level, we found that the size of the required marriage bonus can be quite large. It is an open question whether these predictions are robust to the introduction of some additional realistic features such as labor supply and fertility decisions, *ex ante* heterogeneity, etc., or to an explicit consideration of cohabitation in the design of optimal taxes. We leave these extensions for future research.

## Acknowledgments

We are grateful to Rob Shimer (Associate Editor) and an anonymous referee for their helpful comments and suggestions, and we also thank David Andolfatto, Andrés Erosa, Lutz Hendricks, John Knowles, Peter Morgan, Chris Robinson, Shanon Seitz, Alan Slivinski, Jeffrey Smith, Neil Wallace, Randall Wright, and seminar participants at Centro de Investigación Económica (ITAM) and at the University of Western Ontario for detailed comments on earlier versions of this paper.

## Appendix A

**Proof of Proposition 1.** The equilibrium conditions  $\xi_m = -\mu'_f(\theta_f^*)$ ,  $\xi_f = -\mu'_m(\theta_m^*)$ ,  $\delta_m = \lambda_f$ , and  $\delta_f = \lambda_m$ , along with the reservation property of the optimal strategies, allow us to rewrite (1) as follows:

$$U_m = \frac{(1-t^S)w_m + \pi\mu'_f\mu'_m k(1-t^M)(w_m + w_f) - \pi\mu'_f \int_{\theta_m^*}^{\bar{\theta}} \theta_m dG_m(\theta_m)}{(r + \delta_m)(1 + \pi\mu'_f\mu'_m)}.$$

Hence, men solve

$$\max_{\theta_m^* \geq 0} U_m.$$

Similarly, women solve

$$\max_{\theta_f^* \geq 0} U_f.$$

The existence of search frictions allows us to ignore the constraints  $\theta_m^* \leq \bar{\theta}$  and  $\theta_f^* \leq \bar{\theta}$ , for they will never bind. The Kuhn–Tucker conditions of these problems reveal, after some manipulation, that a matching equilibrium must satisfy the following conditions:

$$k(1-t^M)(w_m + w_f) + \theta_m^* - (1-t^S)w_m + \pi\mu'_f\mu_m \geq 0, \quad (25)$$

$$\theta_m^* \geq 0, \quad (26)$$

with complementary slackness; and

$$(1-k)(1-t^M)(w_m + w_f) + \theta_f^* - (1-t^S)w_f + \pi\mu'_m\mu_f \geq 0, \quad (27)$$

$$\theta_f^* \geq 0, \quad (28)$$

with complementary slackness.

Given condition (b), there cannot be an equilibrium with  $\theta_m^* = \theta_f^* = 0$ . For if  $\theta_f^* = 0$ , then the optimal response for men is to set  $\theta_m^* > 0$ , and vice versa. Thus, there are only three possible cases:

- (i) both thresholds are positive;
- (ii) only  $\theta_m^*$  is positive;
- (iii) only  $\theta_f^*$  is positive.

Existence of a solution then follows as in Proposition 1 in Burdett and Wright (1998). Moreover, it is easy to show that, under log-concavity, the absolute value of the inverse of the slope of the reaction function of women  $\theta_f^* = r_f(\theta_m^*)$  (given by (27)–(28)) is greater than the slope of the reaction function of men  $\theta_m^* = r_m(\theta_f^*)$  (given by (25)–(26)) when they intersect. Hence, the equilibrium is unique.  $\square$

**Proof of Proposition 2.** We first show that  $\theta_m^*$  and  $\theta_f^*$  cannot both decrease when  $t^M$  increases. Since the denominator of (8)–(9) is positive, it is enough to show that the numerators cannot be both negative. We can rewrite them respectively as

$$k[a(k) + b(k)] - b(k) \quad (29)$$

and

$$a(k) - k[a(k) + c(k)],$$

where  $a(k) \equiv 1 + \pi\mu'_f\mu'_m$ ,  $b(k) \equiv \pi\mu_m\mu''_f$ , and  $c(k) \equiv \pi\mu_f\mu''_m$  (these expressions depend on  $k$  through  $\theta_f^*$  and  $\theta_m^*$ ). If (29) is negative, then it must be the case that  $k < b(k)/(a(k) + b(k))$ ; therefore,

$$\begin{aligned} a(k) - k[a(k) + c(k)] &> a(k) - \frac{b(k)[a(k) + c(k)]}{a(k) + b(k)} \\ &= \frac{(1 + \pi\mu'_f\mu'_m)^2 - \pi^2\mu''_f\mu_m\mu''_m\mu_f}{a(k) + b(k)} > 0 \end{aligned}$$

given the log-concavity of  $\mu_f$  and  $\mu_m$  and the fact that  $a(k) + b(k) > 0$ . Thus, when  $t^M$  increases then either

- (i) both  $\theta_m^*$  and  $\theta_f^*$  increase, or
- (ii)  $\theta_m^*$  increases and  $\theta_f^*$  decreases, or
- (iii)  $\theta_m^*$  decreases and  $\theta_f^*$  increases. The analysis of the symmetric case shows that case (i) is nonempty.

Consider case (ii). It is straightforward to show the following results: (a)  $\partial\theta_m^*/\partial k < 0$ ,  $\partial\theta_f^*/\partial k > 0$ ,  $\lim_{k \rightarrow 1} \theta_f^* < \bar{\theta}$ ; (b)  $\lim_{k \rightarrow 1} h(k) = h(1) > 0$ , where  $h(k)$  is the denominator of (8)–(9); c)  $\lim_{k \rightarrow 1} a(k) = a(1) > 0$  and  $\lim_{k \rightarrow 1} c(k) = c(1) > 0$ . Therefore,

$$\begin{aligned} \lim_{k \rightarrow 1} \frac{\partial\theta_m^*}{\partial t^M} &= \frac{a(1)}{h(1)} > 0, \\ \lim_{k \rightarrow 1} \frac{\partial\theta_f^*}{\partial t^M} &= -\frac{c(1)}{h(1)} < 0. \end{aligned}$$

By continuity, for  $k$  sufficiently close to one  $\partial\theta_m^*/\partial t^M > 0$  and  $\partial\theta_f^*/\partial t^M < 0$ , so case (ii) is nonempty. The analysis for case (iii) follows in a similar way by letting  $k$  go to zero.  $\square$

**Proof of Proposition 3.** Using  $t(t^M) = E/2w - \gamma t^M/(r + 2\delta + \gamma)$  and  $\gamma = \beta\mu'^2$ , we can rewrite the planner's problem as follows:

$$\max_{t^M} \frac{\theta^*(t^M) + \left(1 - \frac{E}{2w} - \frac{1}{(1+\pi\mu'^2)}t^M\right)w}{r + \delta}.$$

The first-order condition is

$$\frac{\partial\theta^*}{\partial t^M} - \frac{1}{(1+\pi\mu'^2)}w + \frac{2\pi\mu'\mu''}{(1+\pi\mu'^2)^2}\frac{\partial\theta^*}{\partial t^M}t^M w = 0. \quad (30)$$

Inserting  $\partial\theta^*/\partial t^M = 1/(1+\pi\mu'^2 + \pi\mu''\mu)$  into (30) and manipulating reveals that the optimal solution must satisfy the following equation:

$$t^M = \frac{\mu}{\mu'} \frac{(1+\pi\mu'^2)}{2w},$$

which is clearly negative since  $\mu' < 0$ .  $\square$

**Proof of Proposition 4.** (1) With endogenous match dissolution,  $U_m$  is the same as (1) and  $V_m(\theta_m)$  is given by

$$(r + \delta_m)V_m(\theta_m) = k(1 - t^M)(w_m + w_f) + \theta_m + (\delta_f + \psi)(U_m - V_m(\theta_m)) + \psi E[\max\{\xi_m(V(\theta'_m) - U_m), 0\}], \quad (31)$$

where, with some abuse of notation,  $\xi_m$  is also the probability that the agent's spouse proposes to continue with the match. The optimal strategy is to choose a threshold  $\theta_m^*$  that determines the match formation and separation decisions.

Using (1) to write  $E[\max\{V(\theta_m) - U_m, 0\}] = (r + \delta_m)U_m/(\beta\xi_m) - (1 - t^S)w_m/(\beta\xi_m)$ , substituting this into (31) to obtain an expression for  $V_m(\theta_m)$ , inserting it into (1), and integrating using the threshold strategy  $\theta_m^*$ , we obtain

$$U_m = \frac{(1 - t^S)w_m(r + \delta_m + \delta_f + \psi(1 + \mu'_m\xi_m)) - \beta\xi_m\mu'_m k(1 - t^M)(w_m + w_f) + \beta\xi_m \int_{\theta_m^*}^{\bar{\theta}} \theta_m dG_m}{(r + \delta_m)(r + \delta_m + \delta_f + \psi - \mu'_m(\beta - \psi)\xi_m)}.$$

The problem for men is to find  $\theta_m^*$  that solves

$$\max_{\theta_m^* \geq 0} U_m.$$

Similarly, women solve

$$\max_{\theta_f^* \geq 0} U_f.$$

Using the equilibrium conditions  $\lambda_i = \delta_j$  and  $\xi_i = -\mu_j$ ,  $i, j = m, f$ ,  $i \neq j$ , it follows, after some manipulation, that a matching equilibrium is characterized by a pair  $(\theta_m^*, \theta_f^*)$  that satisfies

$$k(1 - t^M)(w_m + w_f) + \theta_m^* - (1 - t^S)w_m + \frac{(\beta - \psi)\mu'_f(\theta_f^*)}{r + \delta_f + \delta_m + \psi}\mu_m(\theta_m^*) \geq 0, \quad (32)$$

$$\theta_m^* \geq 0, \quad (33)$$

with complementary slackness; and

$$(1 - k)(1 - t^M)(w_m + w_f) + \theta_f^* - (1 - t^S)w_f + \frac{(\beta - \psi)\mu'_m(\theta_m^*)}{r + \delta_f + \delta_m + \psi}\mu_f(\theta_f^*) \geq 0, \quad (34)$$

$$\theta_f^* \geq 0, \quad (35)$$

with complementary slackness.

Given condition (c), there cannot be an equilibrium with  $\theta_m^* = \theta_f^* = 0$ . For if  $\theta_f^* = 0$ , then the optimal response for men is to set  $\theta_m^* > 0$ , and vice versa. Thus, there are only three possible cases: (i) both thresholds are positive; (ii) only  $\theta_m^*$  is positive; (iii) only  $\theta_f^*$  is positive. Existence of a solution then follows as in Proposition 1 in Burdett and Wright (1998). It is easy to show conditions a) and b) imply that the absolute value of the inverse of the slope of the reaction function of women  $\theta_f^* = r_f(\theta_m^*)$  (given by (34)–(35)) is greater than the slope of the reaction function of men  $\theta_m^* = r_m(\theta_f^*)$  (given by (28)–(32)) when they intersect. Hence, the equilibrium is unique.

(2) The expressions for  $\partial\theta_m^*/\partial t^M$  and  $\partial\theta_f^*/\partial t^M$  are the same as (8) and (9), except that  $\phi = (\beta - \psi)/(r + 2\delta + \psi)$  replaces  $\pi$ . Hence, the proof is exactly the same as the proof of Proposition 2.  $\square$

### Proof of Proposition 5.

Using

$$t(t^M) = \frac{E}{2w} - \frac{\gamma}{r + 2\delta + \gamma + \psi(1 - \mu'^2)} t^M \quad \text{and} \quad \gamma = \beta\mu'^2$$

we obtain, after algebraic manipulation, the following first-order condition:

$$\begin{aligned} \frac{\partial\theta^*}{\partial t^M} &\left( 1 - \frac{\psi(\mu''\mu + \mu'^2)}{r + 2\delta + \psi} \right) - \frac{r + 2\delta + \psi(1 - \mu'^2)}{(r + 2\delta + \psi + (\beta - \psi)\mu'^2)w} \\ &+ \frac{2\mu'\mu''\beta(r + 2\delta + \psi)}{(r + 2\delta + \psi + (\beta - \psi)\mu'^2)^2} \frac{\partial\theta^*}{\partial t^M} t^M w = 0. \end{aligned}$$

After inserting

$$\frac{\partial\theta^*}{\partial t^M} = \frac{1}{(1 + \phi\mu'^2 + \phi\mu''\mu)} \quad \text{and} \quad \phi = \frac{\beta - \psi}{r + 2\delta + \psi}$$

into the first-order condition, and manipulating the resulting expression, we obtain that the optimal  $t^M$  satisfies the following equation:

$$t^M = \frac{\mu}{\mu'} \frac{(1 + \phi\mu^2)}{2w},$$

which is clearly negative since  $\mu' < 0$ .  $\square$

*Formal analysis of the model with cohabitation* We now proceed to derive the system of equations that characterize a matching equilibrium  $(\theta_m^C, \theta_m^M, \theta_f^C, \theta_f^M)$ , with  $\theta_m^C < \theta_m^M$  and  $\theta_f^C < \theta_f^M$ .

Consider the decision problem faced by a male in this environment. The values of being single, cohabitating, and married are recursively defined by (19), (20), and (21), respectively. Let us simplify the notation and denote by  $E \max$  the expectation in (19) and (20); notice that  $E \max \geq 0$  and it is positive except for the trivial case in which being single dominates cohabitation and marriage for all values of the match-specific component.

Using (19), (20), and (21), we obtain the following expressions after algebraic manipulation:

$$\begin{aligned} \xi_m^C(V_m^C(\theta_m) - U_m) \\ = \xi_m^C \sigma_m^C ((\beta - \psi)(E \max - k(1 - t^S)(w_m + w_f)) - (1 - t^S)w_m + \theta_m), \end{aligned} \quad (36)$$

$$\begin{aligned} \xi_m^M(V_m^C(\theta_m) - U_m) \\ = \xi_m^M \sigma_m^M (\beta(E \max - k(1 - t^M)(w_m + w_f)) - (1 - t^S)w_m + \theta_m). \end{aligned} \quad (37)$$

We want the optimal strategy to be described by a pair of thresholds  $(\theta_m^C, \theta_m^M)$ , with  $\theta_m^C < \theta_m^M$ , which are determined by  $V_m^C(\theta_m^C) = U_m$  and  $\xi_m^C(V_m^C(\theta_m^M) - U_m) = \xi_m^M \times (V_m^C(\theta_m^M) - U_m)$ , respectively. Figure 2 depicts the type of strategy we are looking for.

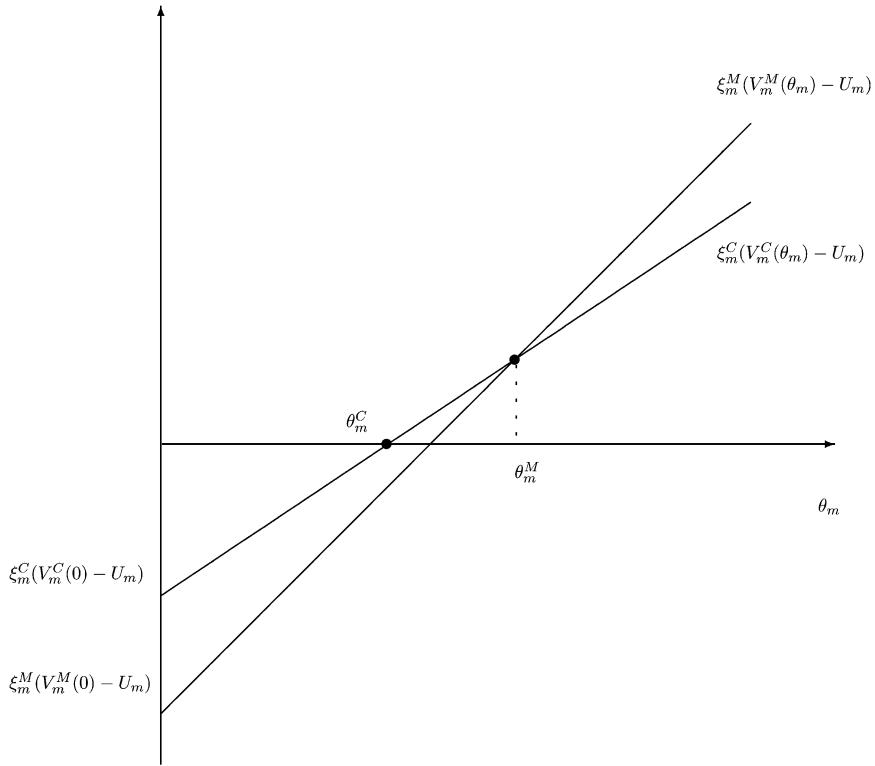


Fig. 2. Male's optimal strategy.

The picture reveals that a necessary condition for the optimal strategy to have this form is that the ‘slope’ of (37) be bigger than the ‘slope’ of (36), or  $\xi_m^M \sigma_m^M > \xi_m^C \sigma_m^C$ . But we also need to make sure that  $\theta_m^C < \theta_m^M$  (i.e., that (36) ‘crosses’ 0 before it intersects (37)). Using  $V_m^C(\theta_m^C) = U_m$ ,  $\xi_m^C(V_m^C(\theta_m^M) - U_m) = \xi_m^M(V_m^C(\theta_m^M) - U_m)$ , and expressions (36) and (37), we obtain, after simple algebraic manipulation,

$$\theta_m^M = \theta_m^C + \frac{\xi_m^M \sigma_m^M (\psi E \max + k(t^M - t^S)(w_m + w_f))}{\xi_m^M \sigma_m^M - \xi_m^C \sigma_m^C}. \quad (38)$$

Thus, if  $\xi_m^M \sigma_m^M > \xi_m^C \sigma_m^C$  and  $t^M \geq t^S$ , then  $\theta_m^C < \theta_m^M$  and the optimal strategy for a male has the desired form. (Notice that, since  $E \max > 0$ ,  $\theta_m^C < \theta_m^M$  also obtains in this case if  $t^M$  is ‘slightly’ less than  $t^S$ . The quantitative exercise provides an illustration of this point.)

Under these assumptions, we can rewrite (19), (20), and (21) as follows:

$$\begin{aligned} & (r + \delta_m)U_m \\ &= (1 - t^S)w_m + \beta \xi_m^C \int_{\theta_m^C}^{\theta_m^M} (V_m^C(\theta_m) - U_m) dG_m(\theta_m) \end{aligned}$$

$$+ \beta \xi_m^M \int_{\theta_m^M}^{\bar{\theta}} (V_m^M(\theta_m) - U_m) dG_m(\theta_m), \quad (39)$$

$$\begin{aligned} V_m^C(\theta_m) \\ = \sigma_m^C (k(1-t^S)(w_m + w_f) + \theta_m) + \sigma_m^C \psi \xi_m^C \int_{\theta_m^C}^{\theta_m^M} (V_m^C(\theta_m) - U_m) dG_m(\theta_m) \\ + \sigma_m^C \psi \xi_m^M \int_{\theta_m^M}^{\bar{\theta}} (V_m^M(\theta_m) - U_m) dG_m(\theta_m) + \sigma_m^C (\lambda_m + \psi) U_m, \end{aligned} \quad (40)$$

$$V_m^M(\theta_m) = \sigma_m^M (k(1-t^M)(w_m + w_f) + \theta_m + \lambda_m U_m). \quad (41)$$

We will use (39), (40), and (41), along with  $V_m^C(\theta_m^C) = U_m$  and  $\xi_m^C(V_m^C(\theta_m^M) - U_m) = \xi_m^M(V_m^C(\theta_m^M) - U_m)$ , to obtain two equations in  $\theta_m^C$  and  $\theta_m^M$ .

Since the algebra is messy, we will break the derivation into several steps:

**Step 1:** Subtracting  $U_m$  from (40) and using  $V_m^C(\theta_m^C) = U_m$ , we obtain

$$\begin{aligned} & \int_{\theta_m^C}^{\theta_m^M} (V_m^C(\theta_m) - U_m) dG_m(\theta_m) \\ &= \sigma_m^C \int_{\theta_m^C}^{\theta_m^M} \theta_m dG_m(\theta_m) - \sigma_m^C \theta_m^C (\mu'_m(\theta_m^M) - \mu'_m(\theta_m^C)). \end{aligned} \quad (42)$$

Similarly, subtracting  $U_m$  from (41) yields

$$\begin{aligned} & \int_{\theta_m^M}^{\bar{\theta}} (V_m^M(\theta_m) - U_m) dG_m(\theta_m) \\ &= \sigma_m^M \int_{\theta_m^M}^{\bar{\theta}} \theta_m dG_m(\theta_m) - \sigma_m^M \mu'_m(\theta_m^M) k(1-t^M)(w_m + w_f) \\ &+ \sigma_m^M \mu'_m(\theta_m^M) (r + \delta) U_m. \end{aligned} \quad (43)$$

**Step 2:** Inserting (42) and (43) into (39) yields

$$\begin{aligned} & (r + \delta_m) U_m \\ &= \frac{w_m(1-t^M) + \beta \xi_m^C \sigma_m^C \int_{\theta_m^C}^{\theta_m^M} \theta_m dG_m(\theta_m) + \beta \xi_m^M \sigma_m^M \int_{\theta_m^M}^{\bar{\theta}} \theta_m dG_m(\theta_m)}{1 - \beta \xi_m^M \sigma_m^M \mu'_m(\theta_m^M)} \end{aligned}$$

$$-\frac{\beta \xi_m^C \sigma_m^C \theta_m^C (\mu'_m(\theta_m^M) - \mu'_m(\theta_m^C)) + \beta \xi_m^M \sigma_m^M \mu'_m(\theta_m^M) k(1-t^M)(w_m + w_f)}{1 - \beta \xi_m^M \sigma_m^M \mu'_m(\theta_m^M)}. \quad (44)$$

**Step 3:** Inserting (42) and (43) into (40), and using  $V_m^C(\theta_m^C) = U_m$ , yields

$$\begin{aligned} & (r + \delta_m)U_m \\ &= \frac{w_m(1-t^S) + \theta_m^C + \psi \xi_m^C \sigma_m^C \int_{\theta_m^C}^{\theta_m^M} \theta_m dG_m(\theta_m) + \psi \xi_m^M \sigma_m^M \int_{\theta_m^M}^{\bar{\theta}_m} \theta_m dG_m(\theta_m)}{1 - \psi \xi_m^M \sigma_m^M \mu'_m(\theta_m^M)} \\ &\quad - \frac{\psi \xi_m^C \sigma_m^C \theta_m^C (\mu'_m(\theta_m^M) - \mu'_m(\theta_m^C)) + \psi \xi_m^M \sigma_m^M \mu'_m(\theta_m^M) k(1-t^M)(w_m + w_f)}{1 - \psi \xi_m^M \sigma_m^M \mu'_m(\theta_m^M)}. \end{aligned} \quad (45)$$

**Step 4:** Since  $V_m^C(\theta_m^C) = U_m$ ,  $V_m^C(\theta_m^M) - U_m = V_m^C(\theta_m^M) - V_m^C(\theta_m^C) = \sigma_m^C(\theta_m^M - \theta_m^C)$ . Using this expression and  $\xi_m^C(V_m^C(\theta_m^M) - U_m) = \xi_m^M(V_m^C(\theta_m^M) - U_m)$ , we obtain

$$(r + \delta_m)U_m = k(1-t^M)(w_m + w_f) + \theta_m^M - \frac{\xi_m^C \sigma_m^C}{\xi_m^M \sigma_m^M} (\theta_m^M - \theta_m^C). \quad (46)$$

**Step 4:** (44), (45), and (46) are three equations in three unknowns:  $(r + \delta_m)U_m$ ,  $\theta_m^C$ , and  $\theta_m^M$ . Eliminating  $(r + \delta_m)U_m$  reduces the system to the following two equations in  $\theta_m^C$  and  $\theta_m^M$

$$\begin{aligned} \theta_m^C &= \frac{(1 - \psi \xi_m^M \sigma_m^M \mu'_m(\theta_m^M))(1-t^S)w_m - (1 - \beta \xi_m^M \sigma_m^M \mu'_m(\theta_m^M))w_m(k)(1-t^S)}{1 + (\beta - \psi)\xi_m^C \sigma_m^C (\mu'_m(\theta_m^M) - \mu'_m(\theta_m^C)) - \beta \xi_m^M \sigma_m^M \mu'_i(\theta_m^M)} \\ &\quad + \frac{(\beta - \psi)(\xi_m^C \sigma_m^C \int_{\theta_m^C}^{\theta_m^M} \theta_m dG_m(\theta_m) + \xi_m^M \sigma_m^M \int_{\theta_m^M}^{\bar{\theta}_m} \theta_m dG_m(\theta_m) - \xi_m^M \sigma_m^M \mu'_m(\theta_m^M)w_m(k)(1-t^S))}{1 + (\beta - \psi)\xi_m^C \sigma_m^C (\mu'_m(\theta_m^M) - \mu'_m(\theta_m^C)) - \beta \xi_m^M \sigma_m^M \mu'_m(\theta_m^M)} \\ \theta_m^M &= \theta_m^C + \frac{\xi_m^M \sigma_m^M (\psi \xi_m^M \sigma_m^M (\mu'_m(\theta_m^M) \theta_m^C + \int_{\theta_m^M}^{\bar{\theta}_m} \theta_m dG_m(\theta_m)) + w_m(k)(t^M - t^S) + \psi \xi_m^C \sigma_m^C \int_{\theta_m^C}^{\theta_m^M} \theta_m dG_m(\theta_m))}{(1 - \psi \xi_m^M \sigma_m^M \mu'_m(\theta_m^M))(\xi_m^M \sigma_m^M - \xi_m^C \sigma_m^C)}, \end{aligned}$$

where  $w_m(k) = k(w_m + w_f)$ .

A similar analysis holds for women. That is, if  $\xi_f^M \sigma_f^M > \xi_f^C \sigma_f^C$  and  $t^M \geq t^S$ , then  $\theta_f^C < \theta_f^M$ , and the optimal strategy for a female has the desired form. These thresholds satisfy a similar system of equations as the one derived above.

In equilibrium, it must be the case that  $\xi_m^C = \mu'_f(\theta_f^M) - \mu'_f(\theta_f^C)$ ;  $\xi_f^C = \mu'_m(\theta_m^M) - \mu'_m(\theta_m^C)$ ;  $\xi_m^M = -\mu'_f(\theta_f^M)$ ,  $\xi_f^M = -\mu'_m(\theta_m^M)$ ,  $\lambda_m = \delta_f$ ; and  $\lambda_f = \delta_m$ . Thus, a matching equilibrium of the kind we are looking for is a solution  $(\theta_m^C, \theta_m^M, \theta_f^C, \theta_f^M)$  to the system of equations that satisfies these conditions and also  $\theta_m^C < \theta_m^M$  and  $\theta_f^C < \theta_f^M$ .

## References

- Alm, J., Whittington, L., 1995a. Income taxes and the marriage decision. *Applied Economics* 27 (1), 25–31.  
 Alm, J., Whittington, L., 1995b. Does the income tax affect marital decisions? *National Tax Journal* 48 (4), 565–572.

- Becker, G., 1973. A Theory of marriage: Part I. *Journal of Political Economy* 81 (4), 813–846.
- Bloch, F., Ryder, H., 2000. Two-sided search, marriages and matchmakers. *International Economic Review* 41 (1), 93–115.
- Bramlett, M., Mosher, W., 2002. Cohabitation, marriage, divorce and remarriage in the United States. National Center for Health Statistics, Vital Health Statistics 23.
- Bumpass, L., Lu, H., 2000. Trends in cohabitation and implications for children's family contexts in the United States. *Population Studies* 54, 29–41.
- Bumpass, L., Sweet, J., 1989. National estimates of cohabitation. *Demography* 26 (4), 615–625.
- Bumpass, L., Cherlin, A., Sweet, J., 1991. The role of cohabitation in declining rates of marriage. *Journal of Marriage and the Family* 53, 913–927.
- Burdett, K., Coles, M., 1997. Marriage and class. *Quarterly Journal of Economics* 112, 141–168.
- Burdett, K., Coles, M., 1999. Long-term partnership formation: marriage and employment. *Economic Journal* 109, 307–334.
- Burdett, K., Wright, R., 1998. Two-sided search with nontransferable utility. *Review of Economic Dynamics* 1 (1), 220–245.
- Chade, H., 2001. Two-sided search and perfect segregation with fixed search costs. *Mathematical Social Sciences* 42, 31–51.
- Chade, H., 2002. Matching with noise and the acceptance curse. Mimeo. Arizona State University.
- Chade, H., Ventura, G., 2002. Marriage and taxes: a two-sided search analysis. *International Economic Review* 43 (3), 955–985.
- Cherlin, A., 1992. Marriage, Divorce and Remarriage, second ed. Harvard University Press, Cambridge, MA.
- Eeckhout, J., 1999. Bilateral search and vertical heterogeneity. *International Economic Review* 40 (4), 869–887.
- Fields, J., Casper, L., 2001. America's families and living arrangements: March 2000. *Current Population Reports P20-537*. US Census Bureau, Washington, DC.
- Lu, X., Mc-Afee, 1996. Matching and expectations in a market with heterogeneous agents. In: Baye, M. (Ed.), In: *Advances in Applied Microeconomics*, vol. 6. JAI Press.
- Morgan, P., 1996. Two-sided search and matching. Mimeo. Department of Economics, SUNY.
- Schoen, R., Standish, N., 2001. The retrenchment from marriage: results from marital status life tables for the United States, 1995. *Population and Development Review* 27 (3), 553–563.
- Sjostrom, D., Walker, M., 1995. The marriage tax and the rate and timing of marriage. *National Tax Journal* 48 (4), 547–558.
- Shimer, R., Smith, L., 2000. Assortative matching and search. *Econometrica* 68, 343–369.
- Smith, L., 1997. The marriage model with search frictions. Mimeo. Department of Economics, MIT.
- Southern, P., 1998. *Augustus*. Routledge, New York.
- US Census Bureau, 2002. *Statistical Abstract of the United States: 2002* (122nd ed.) Washington, DC.
- Whittington, L., Alm, J., 1996. Till death or taxes do us apart: Income taxes and the divorce decision. *Journal of Human Resources* 32 (2), 388–412.