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MAE 598 Applied CFD
Project 2
No Collaboration

Task I
A 240 cm -long pipe with a diameter of 6 cm and five equally-spaced outlets with diameters of 3 cm spaced 40 cm apart was modeled in design modeler and meshed with a mesh size of 5 mm . The outlets were consecutively numbered one through five starting with the outlet closest to the main pipe inlet. Mesh inflation was used in order to allow ANSYS to better-compute the high velocity gradient along the pipe walls due to the no-slip condition imposed for the water flowing through the pipe. The total number of elements was around 450,000, which is just below the maximum that is available with this license, to ensure sufficient resolution. A parabolic inlet velocity profile was imposed, according to the equation
$u=0.1\left(1-\left(\frac{r}{R}\right)^{2}\right) \frac{m}{s} \quad$ where $r=\sqrt{y^{2}+z^{2}}$ is the radial distance from the center of the pipe and $r=D / 2$ is the radius of the pipe.

An x-velocity contour plot was made for the pipe inlet to show the imposed inlet velocity, and the result is shown below.


The flow was analyzed with Fluent, and a total of 300 iterations were used. Mass flow rates of each of the outlet pipes were plotted and monitored to ensure steady-state values were obtained using the built-in mass flow rate calculation capability in Fluent. The results of the mass flow rates of the outlet pipes were imported into MATLAB and are shown below.


All mass flow rates were outward from the pipe and were taken to be positive, even though they were output from fluent as negative values due to the coordinate system. As can be seen from the plot, there is a relatively linear relation in the mass flow rates of the successive outlets, and each outlet has a lower rate of flow than the previous. This is to be expected due to the pressure drop associated with each outlet. To further illustrate the effects of each outlet on the flow in the main pipe, plots of pressure and $x$-velocity along the centerline of the pipe are shown below. It should be noted, that each outlet is spaced 40 cm from each other and the first outlet is 40 cm from the inlet.


As can be seen from these plots, at each outlet there is a static pressure drop, which then recovers to some degree along with the corresponding velocity drop. This shows the relationship between dynamic pressure and static pressure. At every point where the velocity drops due to each outlet, there is a corresponding recovery in static pressure as can be expected according to the Bernoulli equation. The total pressure, however, is seen to drop all along the pipe as can be seen by the downward trend in both the velocity and static pressure along the pipe.

Task II
If a uniform output across all of the outlet pipes is desired, this means that they need to be different sizes. This was calculated by programming a MATLAB script that predicts the size of the outlet that is needed to do so. To write this script, it was assumed that the middle pipe (outlet 3) would remain the same size, and all of the other outlet pipe sizes would be changed to match it. This is because the size of the pipe was anticipated to be midway between the others, and a baseline size was desired so that the pipes did not become too large or too small during the iteration process required to make the flows equal.

First, a fluent output file was created to save the mass flow rates of each outlet to a file, using the builtin mass flow rate calculation capability. This gave the mass flow rates through each of the 5 pipes. The MATLAB code was written to import these files, pick out the final iteration result, and then compare the mass flow rates of the other pipes to the middle one (pipe 3). This gave an output of the ratio of the flow rate of each pipe in relation to the middle one. This data was then used to calculate a new diameter for each pipe, according to the rule that the cross-sectional area would be multiplied by the reciprocal of the output ratio. In other words, if pipe one output 1.2 times the mass flow rate of pipe three, it's area was multiplied by $1 / 1.2$ and the new pipe diameter was calculated from this area. This method was used because the flow rates from the uniform pipe seemed to follow a linear pattern. This seemed to suggest that a simple scheme such as this would be able to hone the flows towards the mean without uneven weighting to different outlets. Also, the pipe cross-sectional area was chosen to be the parameter that was changed according to the relative mass flow rate because of the fact that mass flow rate is defined by the products of the density, cross-sectional area, and velocity of the flow according to:

$$
\dot{m}=\rho A v
$$

This means that the relative mass flow rate will be changed by the same proportion as the crosssectional area as long as the density and velocity through the pipe remain constant. It can be assumed that the density will remain almost constant due to the incompressible nature of liquids, however the velocity could not be regarded as exactly constant. Although, it was unknown how much the velocity would change in relation to the cross-section change, the velocity would likely change because of the change in pressure drop across the pipe. This was assumed to be small enough not to worry about because of the small inlet velocity and low static pressure of the pipe, and it was thought that multiple iterations would be able to work around any differences. It was known that each pipe would affect each other pipe's velocity because of this, but as long as the flow of each pipe got closer to the middle pipe with each successive iteration, this would not be an issue.

The success of this iterative method to finding the outlet pipe sizes needed to achieve a uniform mass flow rate from each outlet was measured according to the " S " index, given by:

$$
S=\frac{1}{M} \sqrt{\frac{1}{5} \sum_{k=1}^{5}\left(m_{k}-M\right)^{2}}
$$

Where

$$
M=\sum_{k=1}^{5} m_{k}
$$

And $m_{k}$ is the mass flow rate of the $k t h$ outlet pipe and $M$ is the mean of the mass flow rate. The " $S$ " value measures the normalized deviation from the mean, so a value of zero would mean all flows are exactly the same. Since this is not possible, a low value of $S$ was desired to show that the flows of all of the pipes were approximately the same. These equations were used to calculate the $S$ value for each iteration in MATLAB from the mass flow outputs of each outlet.

For the uniform pipe size in task one, the $S$ value was calculated to be 0.3154 . This served as a baseline "bad" value that had to be decreased significantly. The goal was to get the $S$ value as low as possible, but a value below 0.01 would be sufficient. The $S$ values for each iteration, as well as the initial value are shown below. With this method, 6 iterations reduced the $S$ index to 0.0013 , and only 4 iterations got the $S$ value below the required value, so this method was deemed to have worked well for the requirements.

| Iteration | S Index Value |
| :---: | :---: |
| 0 <br> (Uniform Pipe Diameter <br> of 0.03m) | 0.3154 |
| 1 | 0.0917 |
| 2 | 0.0307 |
| 3 | 0.0115 |
| 4 | 0.0070 |
| 5 | 0.0026 |
| 6 | 0.0013 |

The values obtained for the diameters of the outlet pipes from the $6^{\text {th }}$ iteration are as shown below along with their flow rates:

| Outlet \# | Outlet 1 | Outlet 2 | Outlet 3 | Outlet 4 | Outlet 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Diameter $(\mathrm{m})$ | 0.023601 | 0.026368 | 0.03 | 0.03596 | 0.04724 |
| Mass Flow <br> $(\mathrm{kg} / \mathrm{s})$ | 0.014165 | 0.014146 | 0.014158 | 0.014113 | 0.014151 |

The mass flow rates of the outlets for the $1^{\text {st }}$ and $6^{\text {th }}$ iteration, as well as the initial uniform pipe size condition as shown in Task I are plotted below. This shows the effects of the iterative process getting closer to uniform mass flow rate with increased numbers of iterations. The final iteration can be seen to be nearly perfectly horizontal which indicates very uniform outlet flow rates.


The centerline $x$-velocity and pressure values of the uniform-output design are plotted below in order to compare this design with the initial uniform pipe-size design.


It can be seen from this plot that the dips in pressure associated with each outlet pipe are much more uniform in the uniform flow case than in the uniform pipe size design. Also, the midline velocity drops are larger on the outlets closer to the inlet for the uniform size design, but with the uniform outlet, the velocity drops off less for the pipes closer to the inlet. This is likely due to the throttling effect the second design has on the pipes closer to the inlet, and the fact that for the later outlets, there is less total pressure to draw from, so a larger velocity drop has to occur to draw the same amount of water.

