MAE 460
Project 1
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| Name of Collaborator: No collaboration | Contribution or collaborative effort |
| :--- | :--- |
| Task(s), specific detail |  |
|  |  |

## Problem 1 Water Heater:

Since the cool inlet water is at 20 C , and the bottom of the water heater is at 55 C , these are the bounds for the temperature in the system, and the operating temperature is chosen to be the midpoint between these two values, 37.5 C(310.65 K).

Using the NIST (Jones) formula for the density of air free water equation 3, (note: this is fitted for temperatures between 5C and 40C, and there may be some error in the temperatures higher than this.)

$$
\begin{equation*}
\rho=999.85308+6.32693 * 10^{-2} T-8.523829 * 10^{-3} T^{2}+6.943248 * 10^{-5} T^{3}-3.82121 * 10^{-7} T^{4} \tag{3}
\end{equation*}
$$

The coefficient of thermal expansion Beta can be found using the derivative of density with respect to temperature, at the operating temperature.

$$
\begin{gathered}
B=-\frac{1}{\rho_{0}} \frac{d \rho}{d T}=6.32693 * 10^{-2}-2 * 8.523829 * 10^{-3} T+3 * 6.943248 * 10^{-5} T^{2}-4 * 3.82121 * 10^{-7} T^{3} \\
B=3.66 * 10^{-4} K^{-1}
\end{gathered}
$$

Using steam tables for saturated liquid water, operating density is $993.04 \mathrm{~kg} / \mathrm{m}^{\wedge} 3$


Figure 1-1: Mesh at Plane of Symmetry
A mesh refinement was made at the bottom of the tank to provide better resolution for the heat flux. There is also automatic inflation applied along the walls and for the smaller diameter pipes.


Figure 1-2: Average Outlet Temperature vs. Iteration g=-9.81 m/s^2

This plot shows the convergence of the average outlet temperature over 1848 iterations. The final value is 297.57 K . The value 100 iterations before this is 297.47 K .


Figure 1-3: Y-Velocity Contour Plot g=-9.81 m/s^2
In the figure, the cold inlet fluid can be seen as having a negative y velocity, as the average temperature in the tank is higher than the inlet fluid. The inlet fluid is then less dense, and experiences a negative buoyancy force. The tank experiences convective currents as the fluid mixes.


Figure 1-4: Temperature Contour Plot $g=-9.81 \mathrm{~m} / \mathrm{s}^{\wedge} 2$
The stream of cool water can be seen heating up as it 'falls', as well as the temperature distribution along the bottom surface.

## Part b

In this section, the gravitational acceleration is lowered to $3.72 \mathrm{~m} / \mathrm{s}^{\wedge} 2$.


Figure 1-5: Average Outlet Temperature vs. Iteration $g=-3.72 \mathrm{~m} / \mathrm{s}^{\wedge} 2$

This plot shows the convergence of the average outlet temperature over 1365 iterations. The final value is 296.82 K . The value 100 iterations before this value is 296.85 K . In this case, the outlet temperature is less than 1 degree lower than that in part (a), which is an $18 \%$ difference in the temperature differential between inlet and outlet.


Figure 1-6: Y -Velocity Contour Plot $\mathrm{g}=-3.72 \mathrm{~m} / \mathrm{s}^{\wedge} 2$

Since there is a smaller negative buoyancy force to the cool inlet water, the water travels further into the tank before it starts flowing downward. It also creates a strong convective motion in the tank as shown by the high positive $y$ velocity on the left of the figure. The cool water stream first gains y velocity then it moves downward.


Figure 1-6: Temperature Contour Plot $\mathrm{g}=-3.72 \mathrm{~m} / \mathrm{s}^{\wedge} 2$
Here, the temperature can be much better seen as heating up as it flows through the tank. The cool stream of inlet water can be seen 'curving' downwards as it enters the tank, but it takes a longer distance into the tank to do so. On the left side of the figure, the high temperature on the side wall comes from the clockwise flow where the water flows over the hot bottom surface.

## Problem 2

In this problem, a helical pipe is configured so that a constant heat flux of $600 \mathrm{~W} / \mathrm{m}^{\wedge} 2$ is added to the fluid. The effect of the inlet velocity of the fluid on the differential in temperature is found.

| Velocity inlet $(\mathrm{m} / \mathrm{s})$ | 0.01 | 0.02 | 0.04 | 0.08 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~T}_{\text {out }}(\mathrm{K})$ | 308.76 | 304.47 | 302.27 | 301.15 |
| $\Delta \mathrm{~T}(\mathrm{~K})$ | 8.76 | 4.47 | 2.27 | 1.15 |

Table 2-1: Average Outlet Temperatures for Varying Inlet Velocities

The values of the average outlet temperature were monitored until a steady state was reached. In this case, 300 iterations were enough for all cases.


Figure 2-1: Plot of Temperature Differential vs Inlet Velocity

The temperature difference decreases as the velocity increases because the fluid has less time to absorb the heat. In the simplified hand calculation this would be a linear relationship.


Figure 2-2: Temperature Plot on the Outlet $\mathrm{V}_{\text {in }}=0.08 \mathrm{~m} / \mathrm{s}$

The temperature at the inner edge is significantly higher than the temperature at the outer edge. This is because the velocity is lower at the inner edge, and has more time to absorb the energy from the wall.


Figure 2-3: Velocity Magnitude on the Outlet $\mathrm{V}_{\text {in }}=0.08 \mathrm{~m} / \mathrm{s}$

The velocity contour diagram confirms the velocity at the inner edge is lower, because the distance the fluid has to travel is less that that at the outer edge because of the difference in the radius.

## Problem 3:



Figure 3-1: Average Outlet Velocity vs Flow Time
As heat is added to the air, the temperature increases and therefore the density decreases which forces air out of the pressure outlet. As expected, the velocity normal to the output increases with time. At about 2.5 s , there is an inflection point where the rate of change in the velocity becomes negative as more air is discharged. The problem was set up with a time step of 0.1 s , and a max of 100 iterations per step.


Figure 3-2: Static Pressure Contour ( $\mathrm{t}=5 \mathrm{~s}$ )

Here is the pressure at the plane of symmetry. There is a pressure differential between the inside and the outside air which is causing the flow. There is also a pressure gradient along the pipe that can be seen.


Figure 3-3: Static Temperature Contour ( $\mathrm{t}=5 \mathrm{~s}$ )
There was a mesh enhancement at the top and bottom of the container to capture the temperature gradient better.


Figure 3-4: Density Contour ( $\mathrm{t}=5 \mathrm{~s}$ )
The density contour looks very similar to the temperature contour, because for an ideal gas density is inversely proportional to temperature.


Figure 3-5: Velocity Normal to Outlet Contour ( $\mathrm{t}=5 \mathrm{~s}$ )

The velocity in the $x$ direction shows that the air is moving out of the container. The air is constricted, and there is a higher velocity along the pipe.

The average density was calculated using a volume average integral and was P_bar=1.1826086 kg/m^3. The variation of density across the system at $\mathrm{t}=5 \mathrm{~s}$ is $13.53 \%$

For task 1, where the Boussinesq approximation is used, the variation in density can be calculated as the thermal expansion coefficient multiplied by the variation in temperature across the whole system. The variation across the density is $1.3 \%$.

## References

Jones, Frank E, and Georgia L Harris. "ITS-90 Density of Water Formulation for Volumetric Standards Calibration." Journal of Research of the National Institute of Standards and Technology, vol. 97, no. 3, 1992, p. 335., doi:10.6028/jres.097.013.

