MAE 460/560 Applied CFD, Fall 2022   Homework #2 (4 points)

All tasks are for both MAE460 and MAE560.

Task 1: Compressible flow through a nozzle

This exercise serves as an addendum to Project 1. While a task in that project requires the use of *density based solver* to treat a compressible flow, the system therein has a moderate *Mach number* and does not reach the supersonic regime. In this task, we will simulate two cases of a 2-D flow through a nozzle, one in the supersonic regime and the other in the subsonic regime.

Task 1a: Supersonic case

Consider a 2-D flow through a nozzle as illustrated in Fig. 1 (not drawn to scale). The system is symmetric with respect to the *x*-axis. The profile of the wall of the nozzle is given by

$$F(x) = 0.3 + 0.1 \, [\, tanh(10 \, x - 13) - tanh(10 \, x - 7) \,] , \quad 0 \le x \le 2 , \qquad \text{Eq. (1)}$$

as shown in Fig. 2. <u>The unit of *x* and *F*(*x*) is meter</u>. The system is filled with air. A higher pressure is imposed at the left opening and a lower pressure at the right opening. The pressure difference drives the flow through the nozzle. Perform a simulation with the following setting:

(1) Select *Density based* solver and seek *steady* solution.
(2) Set the density of air to *ideal gas* to allow density to vary with pressure and temperature. Turn *Energy equation* on.
(3) Select *Inviscid* model.
(4) Set *Operating pressure* to 0. (This means the values of pressure imposed at the inlet and outlet are those of absolute pressure.)
(5) Set the left opening as a *pressure inlet* and impose
    (i) *Gauge total (stagnation) pressure* = 100000 Pa
    (ii) *Supersonic/Initial gauge pressure* = 98000 Pa
    (iii) *Total temperature* = 500°K.
(6) Set the right opening as a *pressure outlet* and impose *Gauge pressure* = 5000 Pa. Set the temperature of backflow to 200°K.
(7) For *Reference values*, choose "*Compute from pressure inlet*".
(8) For *Initialization*, choose "Standard" and "*Compute from pressure inlet*".

Since this is an inviscid simulation, there is no need to place highly concentrated mesh near the wall. In other words, the nodal points can be distributed more uniformly in the direction normal to the wall. You may use full-nozzle or half-nozzle geometry, the latter by invoking symmetry.

From the steady solution, the deliverables are:

(D1) Contour plots of *x-velocity*, *density*, and *static temperature*.
(D2) Line plots of *x-velocity* and *Mach number* along the *x*-axis (i.e., the axis of symmetry of the nozzle).

Task 1b: Subsonic case

Repeat Task 1a but change the boundary conditions to the following:

(5) Set the left opening as a *pressure inlet* and impose
     (i) *Gauge total (stagnation) pressure* = 100000 Pa
     (ii) *Supersonic/Initial gauge pressure* = 99900 Pa
     (iii) *Total temperature* = 500°K.
(6) Set the right opening as a *pressure outlet* and impose *Gauge pressure* = 99500 Pa. Set the temperature of backflow to 500°K.

The deliverables are:
(D3) Contour plots of *x-velocity*, *density*, and *static temperature*.
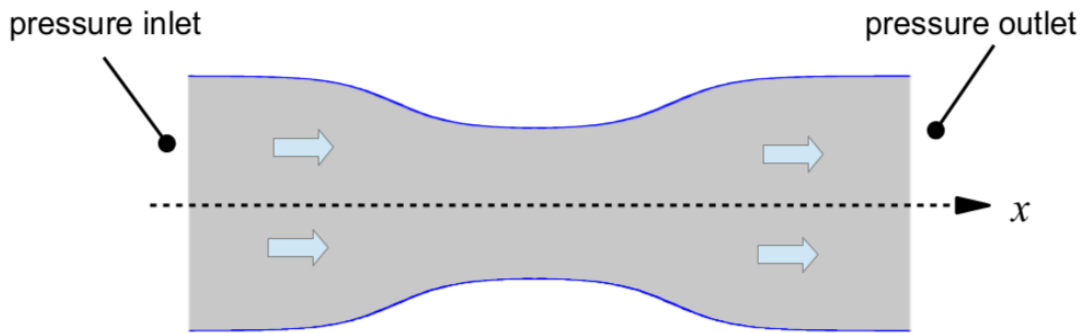(D4) Line plots of *x-velocity* and *Mach number* along the *x*-axis.

Fig. 1 A sketch of the 2-D nozzle used in Task 1. Not drawn to scale.
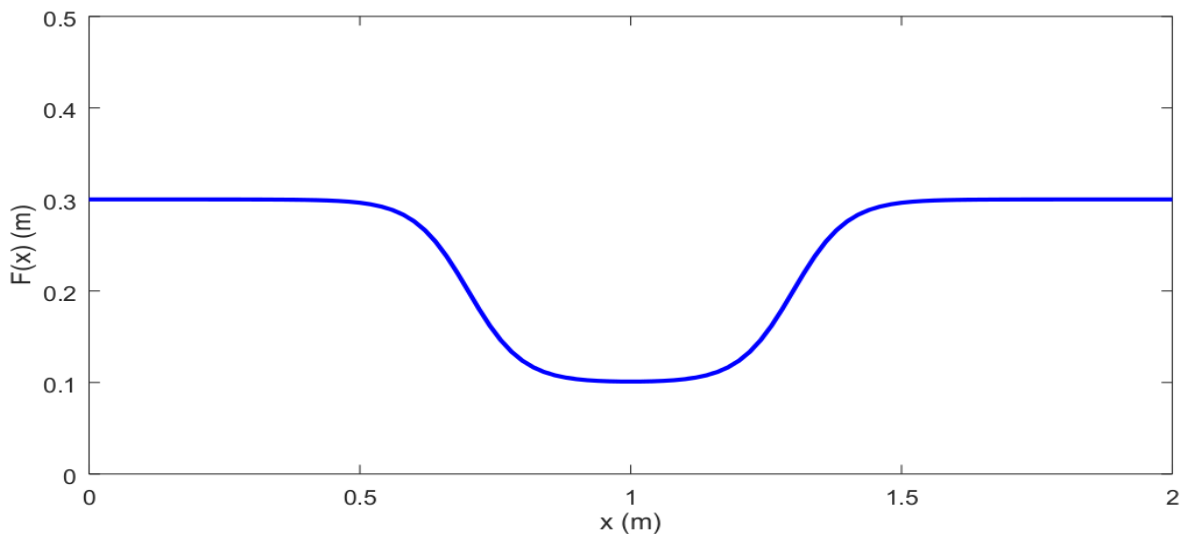

Fig. 2 The profile of the wall of the nozzle. The abscissa is the *x*-axis. The nozzle is 2 m long. [Note that the half-width of the nozzle is not exactly 0.3 at $x = 0$ and 2. Please use the formula in Eq. (1) to ensure that there are no gaps in the geometry.]