

### Task 1, 2, and 3

See reference solutions.

### Task 4

For this task, it is slightly tricky to set the optimal mesh size ( $\Delta x$ ) and time step size ( $\Delta t$ ) given the constraint of 500k elements (and limit of computing power for a typical laptop). One of the reasons is the contrast of high velocity in the drainage pipe vs. much lower velocity in the basin. (Yet, we can only choose one  $\Delta t$  for the entire domain.) Also, the difference in the draining time between water and engine oil is due to the difference in viscosity, which acts to retard the downward velocity through the pipe. As such, the mesh resolution in the pipe should be sufficiently high to resolve this effect. Given this background, perhaps the priority is to first set appropriate ( $\Delta x$ ,  $\Delta t$ ) in the pipe (to resolve the high velocity and effect of viscosity on it), then inflate  $\Delta x$  to larger values into the interior of the basin.

In reference solution #1, ( $\Delta x$ ,  $\Delta t$ ) are set to (0.8 cm, 0.05s) and the draining times obtained from the simulations are ( $t_A = 7.2s$ ,  $t_B = 11.5s$ ). This is close to the best we can get, given the constraints discussed above. As pointed out by the author of reference solution #1, if we keep the same  $\Delta x$  but coarsen  $\Delta t$  to 0.1s, the draining times increase to ( $t_A = 8.1s$ ,  $t_B = 12.2s$ ). (This is also corroborated by the result in reference solution #2.) Assuming that the choice of  $\Delta x$  is appropriate, draining time gets longer because the high-speed flow through the pipe is not as accurately resolved with an increased  $\Delta t$ .

Some students have produced a shorter draining time by using  $\Delta t = 0.1s$  or  $0.05s$  but a much coarser mesh resolution (e.g.,  $\Delta x = 5$  cm). This result is likely spurious: The coarse spatial resolution dilutes the effect of viscosity, rendering the flow velocity in the pipe artificially high, thereby reducing the draining time.

Lastly, since it is required that the values of  $t_A$  and  $t_B$  be reported to 0.1s accuracy, the maximum acceptable value of  $\Delta t$  is 0.1s. (A lower value is recommended.) Those who used  $\Delta t > 0.1$  to perform the simulation will receive a deduction.