Task 2

separation of var on the PDE + first 2 b.c.'s:

$\rightarrow G'' = cG \quad G'(0)=0,\ G'(1)=0 \Rightarrow c = 0,\ -(n\pi)^2,\ n=1,2,3,\ldots$

$G_0(x) = 1,\quad G_n(x) = \cos(n\pi x),\ n=1,2,3,\ldots$

$\rightarrow \ddot{H}_n = [-(n\pi)^2 + \pi^2]H_n \Rightarrow$ For $n=0$, $\ddot{H}_0 = \pi^2 H_0$

$\Rightarrow H_0(t) = A_0\cosh(\pi t) + B_0\sinh(\pi t)$

(No need to consider $n>2$, due to the simple b.c.s)

For $n=1$, $\ddot{H}_1 = 0 \Rightarrow H_1(t) = A_1 t + B_1$

For $n=2$, $\ddot{H}_2 = -3\pi^2 H_2$

$\Rightarrow H_2(t) = A_2\cos(\sqrt{3}\pi t) + B_2\sin(\sqrt{3}\pi t)$

$\rightarrow u(x,t) = A_0\cosh(\pi t) + B_0\sinh(\pi t)$
$+ [A_1 t + B_1]\cos(\pi x)$
$+ [A_2\cos(\sqrt{3}\pi t) + B_2\sin(\sqrt{3}\pi t)]\cos(2\pi x)$

also, $u_t(x,t) = \pi A_0\sinh(\pi t) + \pi B_0\cosh(\pi t) + A_1\cos(\pi x)$
$- \sqrt{3}\pi A_2\sin(\sqrt{3}\pi t)\cos(2\pi x) + \sqrt{3}\pi B_2\cos(\sqrt{3}\pi t)\cos(2\pi x)$

b.c. III: $u(x,0) = 3 + \cos(2\pi x)$

$\Rightarrow A_0 + B_0\cos(\pi x) + A_2\cos(2\pi x) = 3 + \cos(2\pi x)$

$\Rightarrow \boxed{A_0 = 3},\ \boxed{B_1 = 0},\ \boxed{A_2 = 1}$

b.c. IV: $u_t(x,0) = 4 + \cos(\pi x)$

$\Rightarrow \pi B_0 + A_1\cos(\pi x) + \sqrt{3}\pi B_2\cos(2\pi x) = 4 + \cos(\pi x)$

$\Rightarrow \boxed{B_0 = \frac{4}{\pi}},\ \boxed{A_1 = 1},\ \boxed{B_2 = 0}$

Full solution:

$$u(x,t) = 3\cosh(\pi t) + \frac{4}{\pi}\sinh(\pi t) + t\cos(\pi x) + \cos(\sqrt{3}\pi t)\cos(2\pi x)$$

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