MAE 384 Homework #1 Fall 2009

Always <u>show your procedure</u>

1 point = 1 percent of your total score for the semester

1. You are given a toy calculator that has only one function of multiplying two numbers, both are restricted to be between 0.000 and 0.999. Moreover, after performing the calculation, the machine will retain only 3 digits (to the right of the floating point) as its outcome. For instance, given A = 0.318, the precise value of A × A should be 0.101124 while the calculator will produce 0.101. For the calculation of A × A × A, the process will unfold as the following

A = 0.318
A × A = 0.101124 ⇒ calculator retains <u>0.101</u>
A × A × A = (A × A) × A = <u>0.101</u> × 0.318 = 0.032118 ⇒ calculator retains <u>0.032</u> as final answer

The underlined numbers are those that have been trimmed by the calculator. Note that the exact value of A × A × A is 0.032157432.

Using this toy calculator, and given A = 0.983, evaluate A^2, A^3, A^4, ..., to A^{10}. Compare the results with those evaluated by using a real calculator (or Matlab). Treat the latter as the "true" values to evaluate the "true relative error" (*cf*. Eq. 1.17 in textbook) produced by the toy calculator. Plot the error as a function of N, the exponent of A (e.g., N = 4 for A^4). <u>Discuss your results</u>. **(1.5 points)**

[For the example given above with A = 0.318, the true relative errors are 0, 0.12%, and 0.48% for N = 1, 2, and 3.]

2. Find the binary representation of the number 9503 (in decimal representation). **(0.5 point)**

3. Find all of the <u>positive</u> solution(s) (ignore zero and negative ones) for the equation,

$$\sin x = 0.4\, x$$

using the bisection method. A numerical solution will be considered satisfactory if it is within ± 0.05 of the exact solution. (Note: There could be one or many solutions. This is for you to find out.) **(2 points)**

4. Find all solutions for the equation

$$e^{-x} - x^2 + 3x = 2$$

using Newton's method. A numerical solution, x_N, will be considered satisfactory if $|x_N - x_{N-1}| < 0.01$, where x_N is the solution after the N-th iteration. There are more than one solutions for this equation. <u>Discuss how different choices of the initial guess lead to different solutions</u>. **(3 points)**