

MAE 384 Homework #1 Fall 2009

Always show your procedure  
**1 point = 1 percent of your total score for the semester**

1. You are given a toy calculator that has only one function of multiplying two numbers, both are restricted to be between 0.000 and 0.999. Moreover, after performing the calculation, the machine will retain only 3 digits (to the right of the floating point) as its outcome. For instance, given  $A = 0.318$ , the precise value of  $A \times A$  should be 0.101124 while the calculator will produce 0.101. For the calculation of  $A \times A \times A$ , the process will unfold as the following

$$A = 0.318$$

$$A \times A = 0.101124 \Rightarrow \text{calculator retains } \underline{0.101}$$

$$A \times A \times A = (A \times A) \times A = \underline{0.101} \times 0.318 = 0.032118 \Rightarrow \text{calculator retains } \underline{0.032} \text{ as final answer}$$

The underlined numbers are those that have been trimmed by the calculator. Note that the exact value of  $A \times A \times A$  is 0.032157432.

Using this toy calculator, and given  $A = 0.983$ , evaluate  $A^2, A^3, A^4, \dots$ , to  $A^{10}$ . Compare the results with those evaluated by using a real calculator (or Matlab). Treat the latter as the "true" values to evaluate the "true relative error" (cf. Eq. 1.17 in textbook) produced by the toy calculator. Plot the error as a function of  $N$ , the exponent of  $A$  (e.g.,  $N = 4$  for  $A^4$ ). Discuss your results. **(1.5 points)**

[For the example given above with  $A = 0.318$ , the true relative errors are 0, 0.12%, and 0.48% for  $N = 1, 2$ , and 3.]

2. Find the binary representation of the number 9503 (in decimal representation). **(0.5 point)**

3. Find all of the positive solution(s) (ignore zero and negative ones) for the equation,

$$\sin x = 0.4 x$$

using the bisection method. A numerical solution will be considered satisfactory if it is within  $\pm 0.05$  of the exact solution. (Note: There could be one or many solutions. This is for you to find out.) **(2 points)**

4. Find all solutions for the equation

$$e^{-x} - x^2 + 3x = 2$$

using Newton's method. A numerical solution,  $x_N$ , will be considered satisfactory if  $|x_N - x_{N-1}| < 0.01$ , where  $x_N$  is the solution after the  $N$ -th iteration. There are more than one solutions for this equation. Discuss how different choices of the initial guess lead to different solutions. **(3 points)**