

MAE 384 Fall 2009 Homework #2

1. The equation,

$$\sin(x) + \cos(x) - 0.2 = 0 ,$$

has infinite many solutions. Try to find **(a)** The positive solution that is closest to the origin ($x = 0$), and **(b)** The negative solution that is closest to the origin. Both must be obtained by using the fixed point iterative method with minimum of 5 iterations (the initial guess does not count as one iteration). Show your procedure and explain how you choose the correct " $g(x)$ " in the standard form of " $x = g(x)$ " for the iterative process.

4 points

2. We wish to solve the following system of linear equations

$$\begin{aligned} 6x_1 + 2x_2 + 3x_3 &= 2 \\ 2x_1 + 8x_2 + x_3 &= 8 \\ 2x_1 + x_2 + 6x_3 &= -4 \end{aligned}$$

(a) Find the exact solution by the method of Gauss elimination. **0.5 point**

(b) Find the solution by using the Gauss-Seidel iterative method with an initial guess of $(x_2, x_3) = (0, 0)$. Perform minimum of 3 iterations, i.e., update x_1 , x_2 , and x_3 at least 3 times each. The initial guess does not count as an update. **3 points**

(c) Taking the result from (a) as the true solution, $\mathbf{x}_T \equiv (x_1, x_2, x_3)_{\text{TRUE}}$, and denoting the numerical solution from (b) as $\mathbf{x}_N \equiv (x_1, x_2, x_3)_{\text{GAUSS-SEIDEL}}$, calculate the true relative error, E , defined by

$$E = \frac{\|\mathbf{x}_T - \mathbf{x}_N\|}{\|\mathbf{x}_T\|} ,$$

where " $\| \cdot \|$ " is Euclidean norm (*cf.* Eq. 4.72 in textbook) of a vector, for the outcome of each iteration in **(b)**.

0.5 point