HW3 Prob 1 Solution Thanks to Todd Shiflett

1) a) $y = y_1 P_1 + y_2 P_2 + y_3 P_3 + y_4 P_4$

$$P_1 = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} = \frac{(x-0)(x-3)(x-5)}{(-2-0)(-2-3)(-2-5)}$$

$$= \frac{x^3-8x^2+15x}{(-2)(-5)(-7)} = -\frac{x^3}{70} + \frac{8x^2}{70} - \frac{15x}{70}$$

$$P_2 = \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} = \frac{(x+2)(x-3)(x-5)}{(2)(-3)(-5)} = \frac{(x+2)(x^2-8x+15)}{30}$$

$$P_2 = \frac{x^3-6x^2-x+30}{30} = \frac{x^3}{30} - \frac{x^2}{5} - \frac{x}{30} + 1$$

$$P_3 = \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} = \frac{(x+2)(x)(x-5)}{(3-\overset{5}{-2})(3-0)(3-\overset{-2}{5})} = \frac{x^3-3x^2-10x}{-30}$$

$$P_3 = -\frac{x^3}{30} + \frac{x^2}{10} + \frac{x}{3}$$

$$P_4 = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} = \frac{(x+2)(x)(x-3)}{(7)(5)(2)} = \frac{x^3-x^2-6x}{70}$$

$$P_4 = \frac{x^3}{70} - \frac{x^2}{70} - \frac{6}{70}x$$

$y = y_1 P_1 + y_2 P_2 + y_3 P_3 + y_4 P_4$

$$y = .58\left(-\frac{x^3}{70} + \frac{8x^2}{70} - \frac{15x}{70}\right) + 1.8\left(\frac{x^3}{30} - \frac{x^2}{5} - \frac{x}{30} + 1\right) + 1.23\left(-\frac{x^3}{30} + \frac{x^2}{10} + \frac{x}{3}\right) + 2.05\left(\frac{x^3}{70} - \frac{x^2}{70} - \frac{6}{70}x\right)$$

$$y = .04x^3 - .2x^2 + .05x + 1.8$$ ✓

At $x = 4$

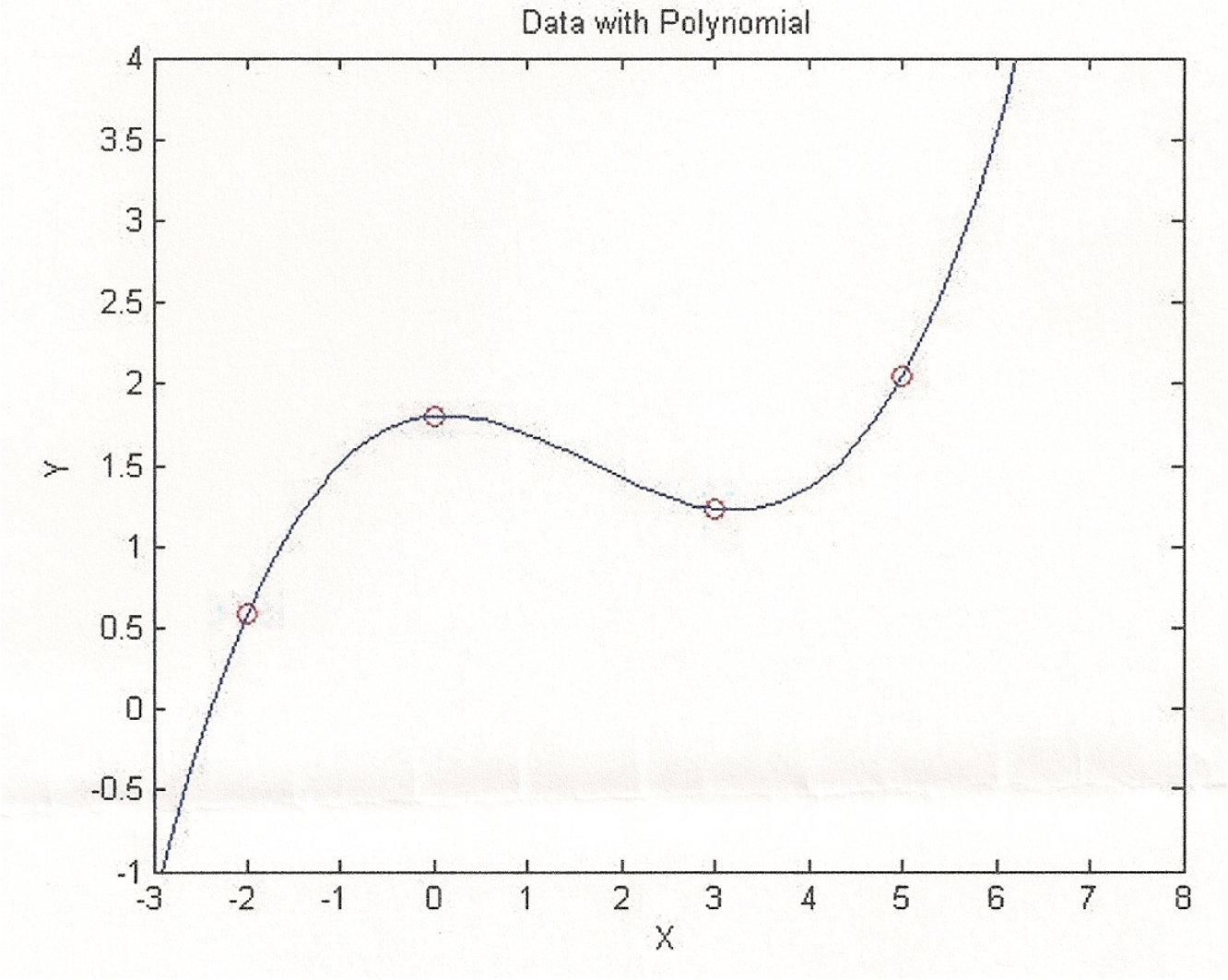
$$y = .04(4)^3 - .2(4)^2 + .05(4) + 1.8$$

$y = 1.36$ at $x = 4$

(Plots continued on next two pages)

(Continued) First plot for HW3 Prob 1

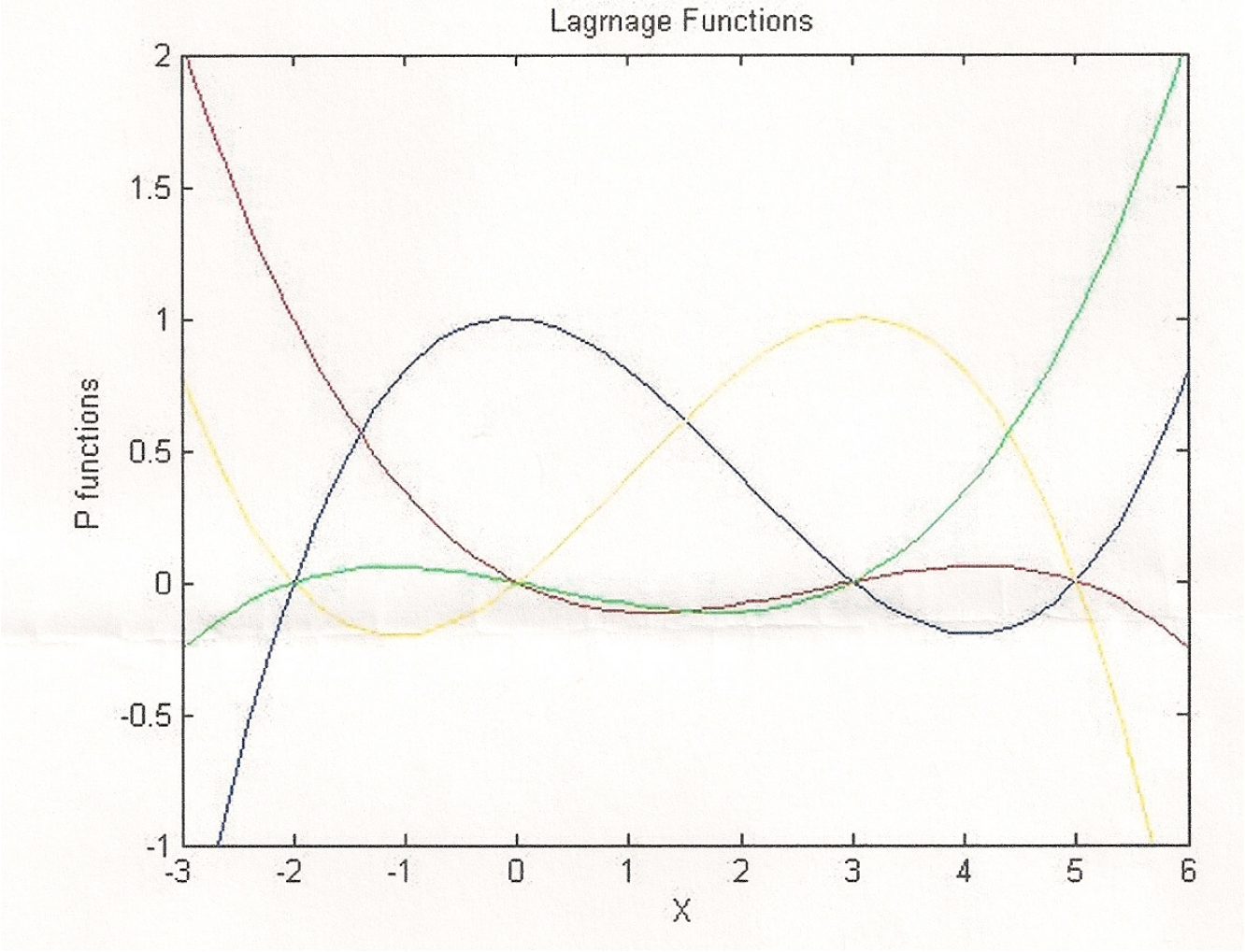
```
>> xdata=[-2 0 3 5]; ydata=[.58 1.8 1.23 2.05];
>> x=[-3:.1:8];
>> y=.04*x.^3-.2*x.^2+.05*x+1.8;
>> plot(xdata,ydata,'ko',x,y,'b-');axis([-3 8 -1 4])
```



(Continued)

(Continued) Second plot for HW3 Prob 1

```
1)b
x=[-3:.1:8];
P1=-1/70*x.^3+8/70*x.^2-15/70*x;
P2=1/30*x.^3-1/5*x.^2-1/30*x+1;
P3=-1/30*x.^3+1/10*x.^2+1/3*x;
P4=1/70*x.^3-1/70*x.^2-6/70*x;
plot(x,P1,'r-',x,P2,'b-',x,P3,'y-',x,P4,'g-'); axis([-3 6 -1 2])
```



HW3 Prob 2 Solution Thanks to Eric Hannoush (See further remark at bottom of next page)

2. $(x_i, y_i) = (1, 3.9), (2, 2.2), (4, 2.0), (6, 3.0)$ ----> 3 splines running through four points

Nine Unknowns: $a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$

Nine equations:

(1) $y''(x_1, y_1) = 0 \rightarrow a_1 = 0$
(2) $y_1 = a_1 x_1^{\,2} + b_1 x_1 + c_1$
(3) $y_2 = a_1 x_2^{\,2} + b_1 x_2 + c_1$
(4) $y_I' - y_{II}' = 0 \rightarrow 2a_1 x_2 + b_1 - 2a_2 x_2 - b_2 = 0$
(5) $y_2 = a_2 x_2^{\,2} + b_2 x_2 + c_2$
(6) $y_3 = a_2 x_3^{\,2} + b_2 x_3 + c_2$
(7) $y_{II}' - y_{III}' = 0 \rightarrow 2a_2 x_3 + b_2 - 2a_3 x_3 - b_3 = 0$
(8) $y_3 = a_3 x_3^{\,2} + b_3 x_3 + c_3$
(9) $y_4 = a_3 x_4^{\,2} + b_3 x_4 + c_3$

```
>> A = [1 0 0 0 0 0 0 0 0; 1 1 1 0 0 0 0 0 0; 4 2 1 0 0 0 0 0 0; 4 1 0 -4 -1 0 0 0 0; 0 0 0 4 2 1 0 0 0; 0 0 0 16 4 1 0 0 0; 0 0 0 8 1 0 -8 -1 0; 0 0 0 0 0 0 16 4 1; 0 0 0 0 0 0 36 6 1]
```

A =

1	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0
4	2	1	0	0	0	0	0	0
4	1	0	-4	-1	0	0	0	0
0	0	0	4	2	1	0	0	0
0	0	0	16	4	1	0	0	0
0	0	0	8	1	0	-8	-1	0
0	0	0	0	0	0	16	4	1
0	0	0	0	0	0	36	6	1

```
>> b = [0; 3.9; 2.2; 0; 2.2; 2; 0; 2; 3]
```

b =

0
3.9
2.2
0
2.2
2.0
0
2.0
3.0

```
>> x = A\b
```

x =

0.0000
-1.7000
5.6000
0.8000
-4.9000
8.8000
-0.5000
5.5000
-12.0000

(Continued)

(Continued)

As shown above,

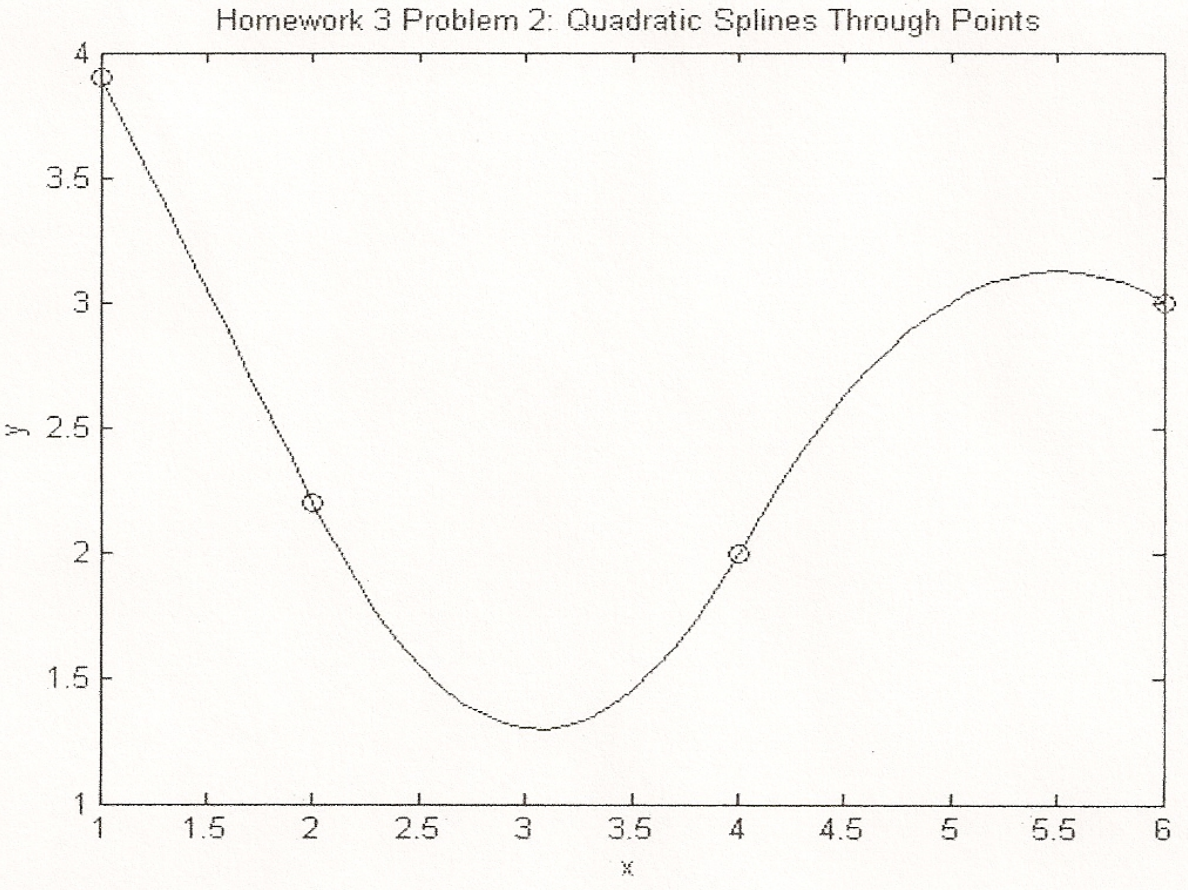
$y = -1.7x + 5.6$; $x_1 \leq x < x_2$

$y = 0.8x^2 - 4.9x + 8.8$; $x_2 \leq x < x_3$

$y = -0.5x^2 + 5.5x - 12$; $x_3 \leq x < x_4$

b.) Plot quadratic splines through the points

```
>> g1 = [1:0.1:2];
>> g2 = [2:0.1:4];
>> g3 = [4:0.1:6];
>> f1 = -1.7.*g1 + 5.6;
>> f2 = 0.8.*g2.^2 -4.9.*g2 + 8.8;
>> f3 = -0.5.*g3.^2 +5.5.*g3 -12;
>> plot(x,y,'ko',g1,f1,'k',g2,f2,'k',g3,f3,'k');
>> xlabel('x');
>> ylabel('y');
>> title('Homework 3 Problem 2: Quadratic Splines Through Points');
```



Note: This is the solution with y" = 0 at x_1. An alternative solution using y" = 0 at x_4 is also acceptable.