MAE384 Fall 2009 Homework #4

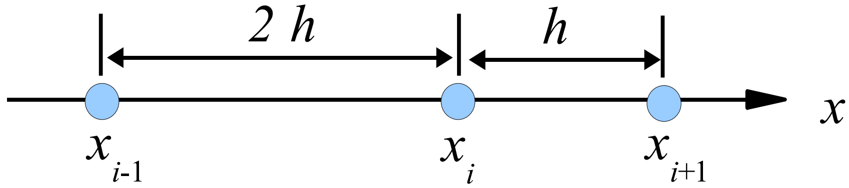
In all problems the argument of a sinusoidal function is in radian.

1. Evaluate the first derivative of the function, f(x) = sin(x^2), using the following methods: **(a)** Obtain f '(x) analytically. **(b)** Numerically evaluate f '(x) using the two-point central difference scheme (3rd formula in Table 6-1 in p. 248) with h = 0.1 and h = 0.05. Plot the results for the interval, $0 \le x \le 5$, against the analytic solution. **(c)** Evaluate f '(x) using the four-point central difference scheme (4th formula in p. 248) with h = 0.1 and h = 0.05. Plot the results for the interval, $0 \le x \le 5$, against the analytic solution. Discuss your results. **(4 points)**

Note: For the case with h = 0.1, you will evaluate f '(x) at x = 0, 0.1, 0.2, 0.3, ..., 4.9, 5. The 51 data points should then be used to make the plot. For h = 0.05, f '(x) will be evaluated at 101 points. Again, those points should be used to make the corresponding plot. To plot the analytic solution, use a finer resolution such as h = 0.01 (501 points for the interval of $0 \le x \le 5$).

2. (Modified from Prob 6.6 in textbook) Derive a three-point finite difference formula for the second derivative, in which f "(x_i) is expressed as a combination of f(x_{i-1}), f(x_i), and f(x_{i+1}), with $x_i - x_{i-1} = 2h$ and $x_{i+1} - x_i = h$. See illustration in Fig. 1. This is an example of a finite difference formula for a non-uniform grid. **(2 points)**

Fig. 1



3. All of the formula in Table 6-1 have a discretization error of $O(h)$, $O(h^2)$, or $O(h^4)$. Try to derive a four-point forward difference formula for the first derivative that has a discretization error of $O(h^3)$. In this formula, f '(x_i) will be expressed as

$$f'(x_i) = \frac{A f(x_i) + B f(x_{i+1}) + C f(x_{i+2}) + D f(x_{i+3})}{h} + O(h^3) \quad .$$

Your goal is to determine A, B, C, and D. (Hint: Try to combine the Taylor series expansion of f(x) at x = x_{i+1}, x_{i+2}, and x_{i+3}.) **(2 points)**

4. Evaluate the integral

$$\int_0^5 \sin(x^2)\, dx \quad ,$$

using the Composite Trapezoidal method for the three cases with h = 0.1, 0.01, and 0.001. **(2 points)**