MAE384 Fall 2009 HW4

Prob 1 Solution   Thanks to Annette Hyla

(1) $f(x) = \sin(x^2)$

(a) $f'(x) = \cos(x^2)(2x)$

(b) Two-point central difference scheme

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} \qquad h = 0.1,\ h = 0.05$$

$h = 0.1$

$$f'(x) = \frac{f(x+0.1) - f(x-0.1)}{2(0.1)}$$

$$f'(x) = \frac{\sin[(x+0.1)^2] - \sin[(x-0.1)^2]}{0.2}$$

$h = 0.05$

$$f'(x) = \frac{f(x+0.05) - f(x-0.05)}{2(0.05)}$$

$$f'(x) = \frac{\sin[(x+0.05)^2] - \sin[(x-0.05)^2]}{0.1}$$

(c) Four-point central difference scheme

$$f'(x_i) = \frac{f(x_{i-2}) - 8f(x_{i-1}) + 8f(x_{i+1}) - f(x_{i+2})}{12h}$$

$h = 0.1$

$$f'(x) = \frac{f(x-0.2) - 8f(x-0.1) + 8f(x+0.1) - f(x+0.2)}{1.2}$$

$$f'(x) = \frac{\sin[(x-0.2)^2] - 8\sin[(x-0.1)^2] + 8\sin[(x+0.1)^2] - \sin[(x+0.2)^2]}{1.2}$$

$h = 0.05$

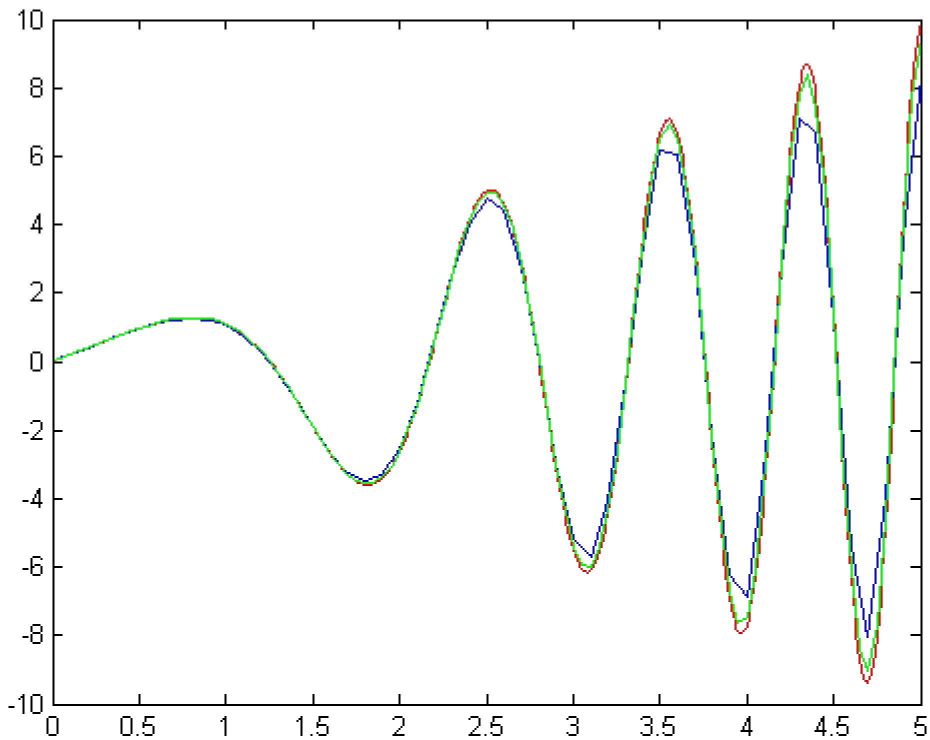
$$f'(x) = \frac{f(x-0.1) - 8f(x-0.05) + 8f(x+0.05) - f(x+0.1)}{0.6}$$

$$f'(x) = \frac{\sin[(x-0.1)^2] - 8[(x-0.05)^2] + 8\sin[(x+0.05)^2] - \sin[(x+0.1)^2]}{0.6}$$

(Matlab codes and plots in next two pages)

(1b)

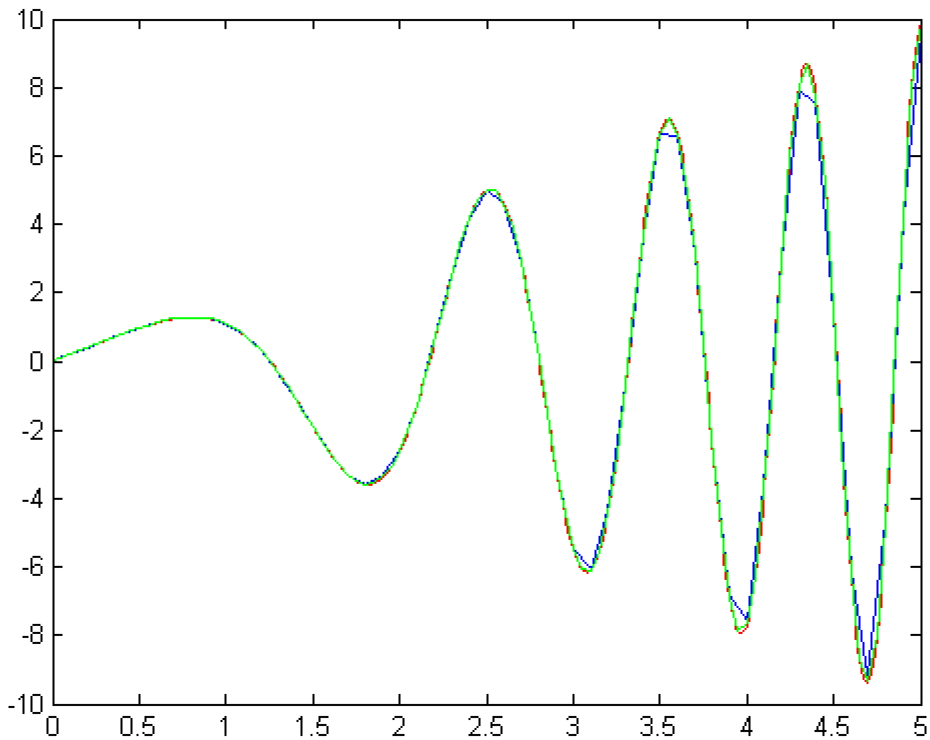
```
>> x1=(0:0.01:5);
>> y1=2*x1.*cos(x1.^2);
>> x2=(0:0.1:5);
>> y2=(sin((x2+0.1).^2)-sin((x2-0.1).^2))/0.2;
>> x3=(0:0.05:5);
>> y3=(sin((x3+0.05).^2)-sin((x3-0.05).^2))/0.1;
>> plot(x1,y1,'r-',x2,y2,'b-',x3,y3,'g-')
```



Red = Analytic   Blue = Numerical, h = 0.1   Green = Numerical, h = 0.05

(1c)

```
>> x1=(0:0.01:5);
>> y1=2*x1.*cos(x1.^2);
>> x2=(0:0.1:5);
>> y2=(sin((x2-0.2).^2)-8*sin((x2-0.1).^2)+8*sin((x2+0.1).^2)-sin((x2+0.2).^2))/1.2;
>> x3=(0:0.05:5);
>> y3=(sin((x3-0.1).^2)-8*sin((x3-0.05).^2)+8*sin((x3+0.05).^2)-sin((x3+0.1).^2))/0.6;
>> plot(x1,y1,'r-',x2,y2,'b-',x3,y3,'g-')
```



Prob 2-4 Solutions   Thanks to Tim Mahoney

Prob 2

2) Start with Taylor series

$$(\mathrm{I})\quad f(x_{i-1}) = f(x_i) - f'(x_i)\,2h + f''(x_i)\,\frac{4h^2}{2}$$

$$(\mathrm{II})\quad f(x_{i+1}) = f(x_i) + f'(x_i)\,h + f''(x_i)\,\frac{h^2}{2}$$

$$1\cdot(\mathrm{I}) + 2\cdot(\mathrm{II}) = \quad f(x_{i-1}) + 2f(x_{i+1}) = 3f(x_i) + 3h^2 f''(x_i) + O(h^3)$$

$$\boxed{f''(x_i) = \frac{f(x_{i-1}) + 2f(x_{i+1}) - 3f(x_i)}{3h^2}} + O(h)$$

Prob 3

3) Start w/ Taylor series

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + (1/2)f''(x_i)h^2 + (1/6)f'''(x_i)h^3$$

$$f(x_{i+2}) = f(x_i) + 2f'(x_i)h + 2f''(x_i)h^2 + (4/3)f'''(x_i)h^3$$

$$f(x_{i+3}) = f(x_i) + 3f'(x_i)h + (9/2)f''(x_i)h^2 + (27/6)f'''(x_i)h^3$$

$$1\cdot 1/6 + A\cdot(4/3) + B(27/6) = 0$$

$$1\cdot 1/2 + A\cdot(2) + B(9/2) = 0$$

$$\begin{pmatrix} 8 & 27 \\ 4 & 9 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \qquad \begin{array}{l} A = -1/2 \\ B = 1/9 \end{array}$$

$$1\cdot f(x_{i+1}) - (1/2)f(x_{i+2}) + (1/9)f(x_{i+3}) = (11/18)f(x_i) + (1/3)f'(x_i)h + O(h^4)$$

$$18f(x_{i+1}) - 9f(x_{i+2}) + 2f(x_{i+3}) = 11f(x_i) + 6h\,f'(x_i)$$

$$\boxed{f'(x_i) = \frac{18f(x_{i+1}) - 9f(x_{i+2}) + 2f(x_{i+3}) - 11f(x_i)}{6h}} + O(h^3)$$

Prob 4

4) $\int_0^5 \sin(x^2)\,dx$

	Ans	
Analytically,	.527917	
Trapezoidal, $h=0.1$,	.536317	$\varepsilon = .0084$
$h=0.01$	.528000	$\varepsilon = .000083$
$h=0.001$	.527918	$\varepsilon = .000001$

See attached MATLAB code

4)
```
%hw4
%prob 4
format long
N=50;  %changes to 500 and 5000
a=0;
b=5;
h=(b-a)/N;
func=inline('sin(x.^2)','x');
x=a:h:b;
for i=1:N+1
    F(i)=func(x(i));
end
I=h*(F(1)+F(N+1))/2+h*sum(F(2:N))
```