MAE384 Fall 2009 HW5 Discussion

Prob 1
The exact value is I = 0.5sin(36) = −0.4959. Numerical integration using h = 0.2 leads to I = −0.0315.
The numerical result improves dramatically to −0.4968 when h is refined to 0.05. This is because the
integrand is a strongly oscillatory function that requires a very small h to properly resolve. The
following are several examples of Matlab codes.

Prob 1, Matlab example #1   Thanks to James Choca

```
f = inline('x*cos(x^2)','x');
a = 0;
b = 6;
h = 0.05;
N = (b-a)/h;
ngridpoint = N+1;
w(1) = 1;
for k = 2:3:N-1
        w(k) = 3;
        w(k+1) = 3;
        w(k+2) = 2;
end
w(ngridpoint) = 1;
for k = 1:ngridpoint
    x(k) = (k-1)*h;
end
i = 0;
for k = 1:ngridpoint
    i = i+f(x(k))*w(k);
end
i = i*3*(h/8);
fprintf('With h=%5.3f, integral = %8.5f \n \n',h,i)
```

This example shows a relatively simple way to assign the weight, w(k). It is not trivial that the
command, "w(ngridpoint) = 1;" is placed below the first loop. This ensures that w(N+1) = 1.
Otherwise, the loop itself would produce w(N+1) = 2.

Prob 1, Matlab example #2   Thanks to Jordana Friedman

```
S1 = 0;
f1 = inline('x*cos(x^2)');
n = 0;
h = 0.2;
for i = 0.2:h:5.8
    if n==2
        S1=S1+2*f1(i);
        n=0;
    else
        S1=S1+3*f1(i);
        n=n+1;
    end
end
I = (f1(0)+S1+f1(6))*(3*h/8)
```

This examples shows an interesting use of the combination of loop and the "if" command. Note that "n" is reset to zero every third time we go through the loop. Also, the index for the loop needs not be an integer. For h = 0.05, change the 4th and 5th lines to "h = 0.05" and "for i = 0.05:h:5.95".
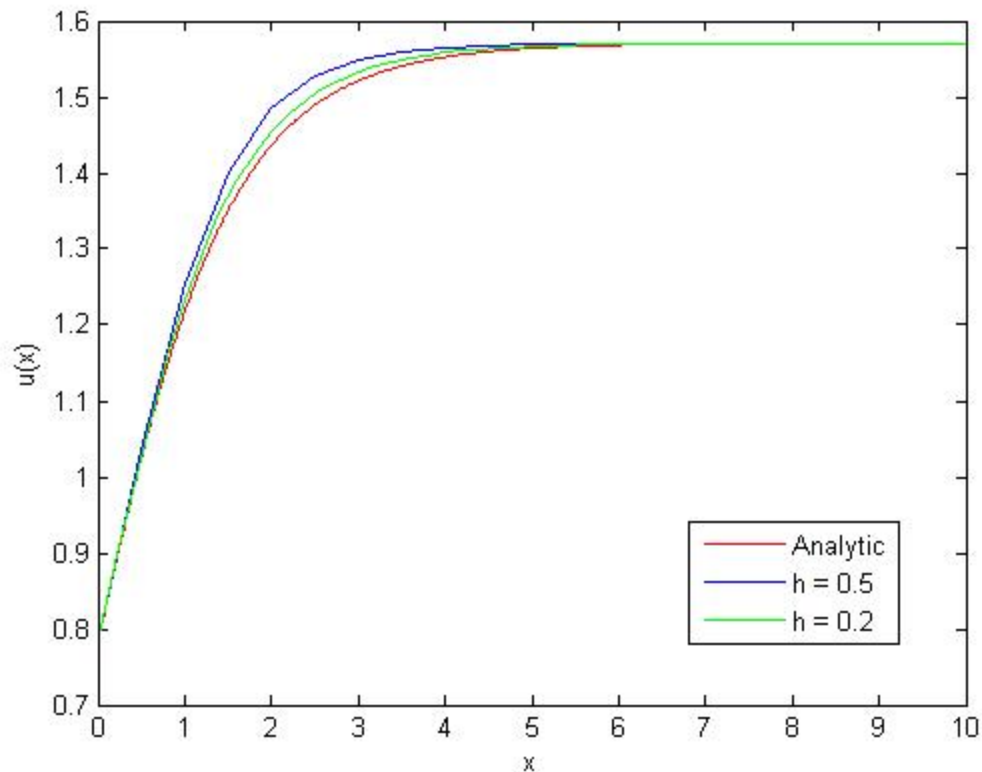

Prob 1, Matlab example #3 (by HPH)

```
h = 0.05;
f = inline('x*cos(x^2)');
I = f(0)+f(6);
for k = 2:(6/h)
    I = I+f((k-1)*h)*(3-mod(mod(k,3),2));
end
I = I*h*(3/8)
```

This example further illustrates the use of the "mod" function.

Prob 2 Solution   Thanks to Joel Johnston

```
clc
clear
h = .5;
t = .2;
f = inline('sin(a)*cos(a)','a');
u(1) = pi/4;
v(1) = pi/4;
    for i = 1:1:20
u(i+1) = u(i) + f(u(i))*h;
    end
    for i = 1:1:50
v(i+1) = v(i) + f(v(i))*t;
    end
x = 0:.001:10;
y = atan(exp(x));
a = 0:.5:10;
s = 0:.2:10;
plot(x,y,'r-',a,u,'b-',s,v,'g-')
```

Prob 3 Solution by hand (prepared by HPH), followed by a Matlab example

$$\overbrace{\frac{du}{dx} = -0.3\,u^2 + 0.1\,x^2 u - 0.5}^{f(x,\,u(x))} \qquad u(1) = 3$$

$h = 0.2$

$$\left\lceil u(x+h) = u(x) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)\,h \right.$$

$$K_1 = f(x,\,u(x))$$
$$K_2 = f(x+h/2,\quad u(x) + K_1 h/2)$$
$$K_3 = f(x+h/2,\quad u(x) + K_2 h/2)$$
$$\left. K_4 = f(x+h,\quad u(x) + K_3 h) \right\rfloor$$

$$u(1.2) = u(1) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)\cdot(0.2)$$

$$K_1 = f(1,\,u(1)) = f(1,\,3) = -0.3\cdot 3^2 + 0.1\cdot 1^2\cdot 3 - 0.5$$
$$= -2.7 + 0.3 - 0.5 = \boxed{-2.9}$$

$$K_2 = f(1 + \tfrac{0.2}{2},\quad u(1) + K_1 \cdot 0.2/2)$$

$$= f(1.1,\quad 3 - (2.9)\cdot(0.2)/2)$$
$$= f(1.1,\quad 2.71) = -0.3\cdot(2.71)^2 + 0.1\cdot(1.1)^2\cdot(2.71) - 0.5$$
$$= -2.20323 + 0.32791 - 0.5$$
$$= \boxed{-2.37532}$$

$$K_3 = f(1 + \tfrac{0.2}{2},\quad u(1) + K_2\cdot(0.2)/2)$$
$$= f(1.1,\quad 3 - 2.37532\cdot(0.2)/2)$$
$$= f(1.1,\quad 2.762468)$$

$$= -0.3\cdot(2.762468)^2 + 0.1\cdot(1.1)^2\cdot(2.762468) - 0.5$$
$$= -2.28936 + 0.334258 - 0.5$$
$$= \boxed{-2.45510}$$

$$K_4 = f(1 + 0.2,\quad u(1) + K_3\cdot(0.2))$$
$$= f(1.2,\quad 2.50898)$$
$$= -0.3\cdot(2.50898)^2 + 0.1\cdot(1.2)^2\cdot(2.50898) - 0.5$$
$$= -1.88849 + 0.36129 - 0.5$$
$$= \boxed{-2.0272}$$
$$u(1.2) = 3 + \frac{1}{6}(-2.9 - 2\cdot(2.37532) - 2\cdot(2.45510) - 2.0272)\cdot(0.2)$$
$$= \boxed{2.5137}$$

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(Continued)

$$u(1.4) = u(1.2) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \cdot (0.2)$$

$$K_1 = f(1.2, u(1.2)) = f(1.2, 2.5137)$$
$$= -0.3 \cdot (2.5137)^2 + 0.1 \cdot (1.2)^2 \cdot (2.5137) - 0.5$$
$$= -1.89560 + 0.36197 - 0.5$$
$$= \boxed{-2.03363}$$

$$K_2 = f(1.2 + 0.1, u(1.2) + (-2.03363) \cdot (0.1))$$
$$= f(1.3, 2.31033)$$
$$= -0.3 \cdot (2.31033)^2 + 0.1 \cdot (1.3)^2 \cdot (2.31033) - 0.5$$
$$= -1.60128 + 0.39044 - 0.5$$
$$= \boxed{-1.71084}$$

$$K_3 = f(1.3, u(1.2) + (-1.71084) \cdot (0.1))$$
$$= f(1.3, 2.34261)$$
$$= -0.3 \cdot (2.34261)^2 + 0.1 \cdot (1.3)^2 \cdot (2.34261) - 0.5$$
$$= -1.64634 + 0.39590 - 0.5$$
$$= \boxed{-1.75044}$$

$$K_4 = f(1.4, u(1.2) + (-1.75044) \cdot (0.2))$$
$$= f(1.4, 2.1636)$$
$$= -0.3 (2.1636)^2 + 0.1 \cdot (1.4)^2 \cdot (2.1636) - 0.5$$
$$= -1.4043 + 0.4240 - 0.5$$
$$= \boxed{-1.4803}$$

$$u(1.4) = u(1.2) + \frac{1}{6}(-2.03363 - 2 \cdot (1.71084) - 2 \cdot (1.75044) - 1.4803) \cdot (0.2)$$

$$= \boxed{2.1658} \quad \leftarrow \text{Final answer.}$$
※

See next page for solution by Matlab.

Prob 3 Solution by Matlab   Thanks to James Choca

```
h = 0.2;
xi = 1;
ui = 3;
f = inline('-0.3*u^2+0.1*x^2*u-0.5','x','u');
x(1) = xi;
u(1) = ui;
for i = 1:2
    k1 = f(x(i),u(i));
    k2 = f(x(i)+0.5*h,u(i)+0.5*k1*h);
    k3 = f(x(i)+0.5*h,u(i)+0.5*k2*h);
    k4 = f(x(i)+h,u(i)+k3*h);
    u(i+1) = u(i)+(1/6)*(k1+2*k2+2*k3+k4)*h;
    x(i+1) = x(i)+h;
end
fprintf('With h = %5.3f, u(1.4) = %8.5f \n',h,u(3))
```

This is actually a very general program that can be used to solve other initial value problems (just change h, xi, ui, and the functional form in the inline command). If we need to go to the next step to obtain u(1.6), simply change "`for i = 1:2`" to "`for i = 1:3`", and so on.