MAE384 Fall 2011 Homework #1

1 point = 1% of your total score for the semester
In all homework and exams, the argument of a sinusoidal function is always in radian, never in degree

1. You are given a toy calculator that has only one function of multiplying two numbers, both are restricted to be between 0.000 and 0.999 with only a 3-digit accuracy. Moreover, after performing the calculation, the machine will retain only 3 digits (to the right of the floating point) as its outcome. For instance, given A = 0.318, the precise value of $A \times A$ should be 0.101124 while the calculator will produce 0.101. For the calculation of $A \times A \times A$, the process will unfold as the following

A = 0.318
$A \times A = 0.101124 \Rightarrow$ calculator retains <u>0.101</u>
$A \times A \times A = (A \times A) \times A =$ <u>0.101</u> $\times 0.318 = 0.032118 \Rightarrow$ calculator retains <u>0.032</u> as final answer

The underlined numbers are those that have been trimmed by the calculator. Note that the exact value of $A \times A \times A$ is 0.032157432. Using this toy calculator, and given A = 0.873, evaluate A^2, A^3, A^4, ..., to A^{10}. Compare the results with those evaluated by using a real calculator (or Matlab). Treat the latter as the "true" values to evaluate the "true relative error" (*cf.* Eq. 1.17 in textbook) produced by the toy calculator. Plot the error as a function of N, the exponent of A (e.g., N = 4 for A^4). <u>Discuss your results</u>. **[1.5 points]**

2. Find the positive solution of the equation

$$\cos(x) = 0.3\,x$$

using the Bisection method with [0, 2] chosen as the initial interval. The solution will be considered satisfactory if its uncertainty (numerical error) is within ± 0.02. **[2.5 points]**

3. **(a)** Given $f(x) \equiv e^{-x} - \sin(x) - 0.2$, find the solutions of $f(x) = 0$ within the interval of $0 < x < 5$ by Newton's method. If there are more than one solutions, find them all. For this problem, a numerical solution, x_N, will be considered satisfactory if $|x_N - x_{N-1}| < 0.01$, where x_N is the solution after the N-th iteration. (The initial guess is x_0.) **[3 points]** **(b)** Discuss how the choice of the initial guess affects the solution. You are encouraged to systematically explore the interval of $0 < x_0 < 5$, but even a few choices of $x_0 = 0.5$, 1.75, 3.0, and 4.7 will be useful. **[1 point]**