

MAE384 Fall 2011 Homework #1

1 point = 1% of your total score for the semester

In all homework and exams, the argument of a sinusoidal function is always in radian, never in degree

1. You are given a toy calculator that has only one function of multiplying two numbers, both are restricted to be between 0.000 and 0.999 with only a 3-digit accuracy. Moreover, after performing the calculation, the machine will retain only 3 digits (to the right of the floating point) as its outcome. For instance, given $A = 0.318$, the precise value of $A \times A$ should be 0.101124 while the calculator will produce 0.101. For the calculation of $A \times A \times A$, the process will unfold as the following

$$A = 0.318$$

$$A \times A = 0.101124 \Rightarrow \text{calculator retains } \underline{0.101}$$

$$A \times A \times A = (A \times A) \times A = \underline{0.101} \times 0.318 = 0.032118 \Rightarrow \text{calculator retains } \underline{0.032} \text{ as final answer}$$

The underlined numbers are those that have been trimmed by the calculator. Note that the exact value of $A \times A \times A$ is 0.032157432. Using this toy calculator, and given $A = 0.873$, evaluate A^2 , A^3 , A^4 , ..., to A^{10} . Compare the results with those evaluated by using a real calculator (or Matlab). Treat the latter as the "true" values to evaluate the "true relative error" (*cf.* Eq. 1.17 in textbook) produced by the toy calculator. Plot the error as a function of N , the exponent of A (e.g., $N = 4$ for A^4). Discuss your results. [1.5 points]

2. Find the positive solution of the equation

$$\cos(x) = 0.3x$$

using the Bisection method with $[0, 2]$ chosen as the initial interval. The solution will be considered satisfactory if its uncertainty (numerical error) is within ± 0.02 . [2.5 points]

3. (a) Given $f(x) \equiv e^{-x} - \sin(x) - 0.2$, find the solutions of $f(x) = 0$ within the interval of $0 < x < 5$ by Newton's method. If there are more than one solutions, find them all. For this problem, a numerical solution, x_N , will be considered satisfactory if $|x_N - x_{N-1}| < 0.01$, where x_N is the solution after the N -th iteration. (The initial guess is x_0 .) [3 points] (b) Discuss how the choice of the initial guess affects the solution. You are encouraged to systematically explore the interval of $0 < x_0 < 5$, but even a few choices of $x_0 = 0.5, 1.75, 3.0$, and 4.7 will be useful. [1 point]