Prob 1 Solution (Thanks to Gerald O'Neill)

1 The set of following 4 pts is given

x	y
-2	0
1	-1.95
3	-1.25
5	-1.75

a) using Lagrange interp method

Lagrange functions

$$L_1 = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} = \frac{(x-1)(x-3)(x-5)}{(-2-1)(-2-3)(-2-5)} =$$

$$= \frac{x^3-9x^2+23x-15}{(-3)(-5)(-7)} = \frac{x^3-9x^2+23x-15}{-105}$$

$$L_2 = \frac{(x+2)(x-3)(x-5)}{(1+2)(1-3)(1-5)} = \frac{x^3-6x^2-x+30}{24}$$

$$L_3 = \frac{(x+2)(x-1)(x-5)}{(3+2)(3-1)(3-5)} = \frac{x^3-4x^2-7x+10}{-20}$$

$$L_4 = \frac{(x+2)(x-1)(x-3)}{(5+2)(5-1)(5-3)} \quad \frac{x^3-2x^2-5x+6}{56}$$

Poly = $\Sigma y \Sigma L$

$$y(x) = y_1L_1 + y_2L_2 + y_3L_3 + y_4L_4$$

$y(x) = -0.05x^3 + 0.3x^2 - 0.2x - 2$ calc at $x=2$

$x=2 \quad y=-1.6$

b) i) plot pts, poly ii) four lagrange functions

i) SEE MATLAB SCRIPT 1, FIG 1

ii) SEE MATLAB SCRIPT 2, FIG 2

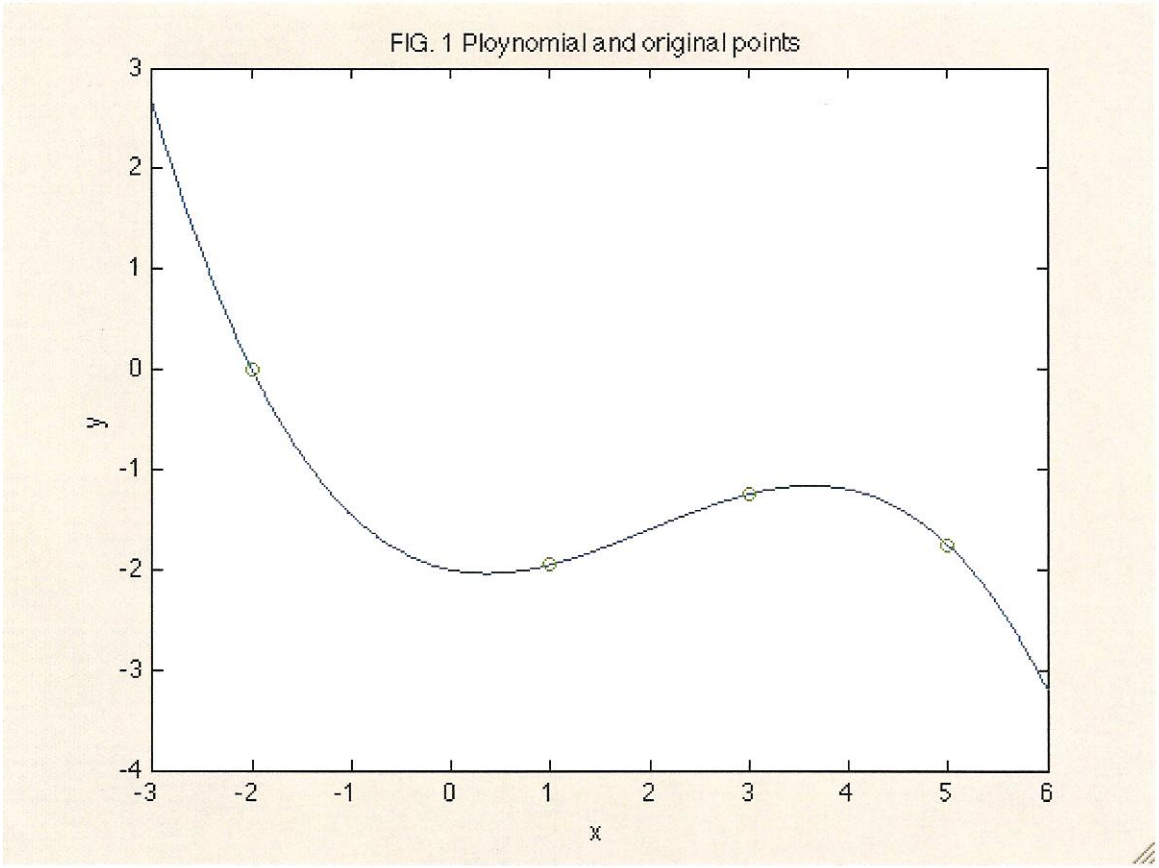
problem 2 on next page

(Matlab code and plots in next 2 pages)

```
clear all
clc

x = [-3:.01:6];
y = (-.05).*x.^3+(.3).*x.^2+(-.2).*x-2;
plot(x,y)
hold on
xo = [-2 1 3 5];
yo = [0 -1.95 -1.25 -1.75];
scatter(xo,yo)
```

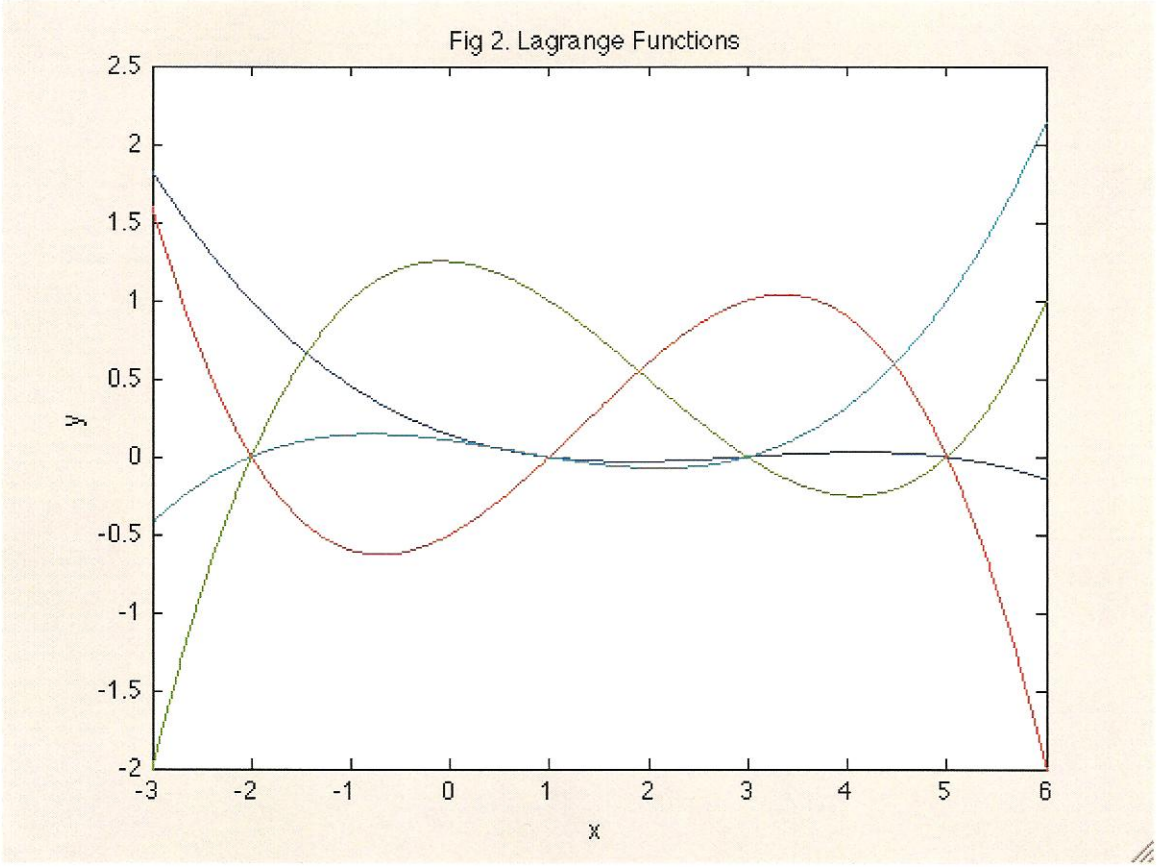
FIG 1



```
clear all
clc

hold off
x = [-3:.01:6];
L1 = (x.^3-9.*x.^2+23.*x-15)/-105;
L2 = (x.^3-6.*x.^2-1.*x+30)/24;
L3 = (x.^3-4.*x.^2-7.*x+10)/-20;
L4 = (x.^3-2.*x.^2-5.*x+6)/56;
plot(x,L1,x,L2,x,L3,x,L4)
```

FIG 2



Note by HPH: The 4 curves shown are the Lagrange functions, L_1, L_2, L_3, and L_4. Some students plotted y_1L_1, y_2L_2, y_3L_3, and y_4L_4, which are also acceptable. In that case, the structure of L_1 will be suppressed since $y_1 = 0$. Nevertheless, one should still plot y_1L_1 (a flat line) to complete the solution.

Prob 2 Solution (Thanks to Gerald O'Neill)

derivatives

$2a_1x_2 + b_1 = 2a_2x_2 + b_2$ $\quad b_1 - 2a_2 - b_2 = 0$

$2a_2x_3 + b_2 = 2a_3x_3 + b_3$ $\quad b_2 + 6a_2 - 6a_3 - b_3 = 0$

$2a_3x_4 + b_3 = 2a_4x_3 + b_4$ $\quad b_3 + 8a_3 - 8a_4 - b_4 = 0$

2 Set 5 pts given

x	y
0	1
1	2
3	2.5
4	1
6	1

$f_1 = b_1x_1 + c_1 = y_1$ $\quad 0 \cdot b_1 + c_1 = 1$

$b_1x_2 + c_1 = y_2$ $\quad 1 \cdot b_1 + c_1 = 2$

$f_2 = a_2x_2^2 + b_2x_2 + c_2 = y_2$ $\quad 1 \cdot a_2 + 1 \cdot b_2 + c_2 = 2$

$a_2x_3^2 + b_2x_3 + c_2 = y_3$ $\quad 9 \cdot a_2 + 3 \cdot b_2 + c_2 = 2.5$

$f_3 = a_3x_3^2 + b_3x_3 + c_3 = y_3$ $\quad 9 \cdot a_3 + 3b_3 + c_3 = 2.5$

$a_3x_4^2 + b_3x_4 + c_3 = y_4$ $\quad 16 \cdot a_3 + 4b_3 + c_3 = 1$

$f_4 = a_4x_4^2 + b_4x_4 + c_4 = y_4$ $\quad 16 \cdot a_4 + 4b_4 + c_4 = 1$

$a_4x_5^2 + b_4x_5 + c_4 = y_5$ $\quad 36 \cdot a_4 + 6b_4 + c_4 = 1$

b_1	c_1	a_2	b_2	c_2	a_3	b_3	c_3	a_4	b_4	c_4				
0	1	0	0	0	0	0	0	0	0	0		b_1		1
1	1	0	0	0	0	0	0	0	0	0		c_1		2
0	0	1	1	1	0	0	0	0	0	0		a_2		2
0	0	9	3	1	0	0	0	0	0	0		b_2		2.5
0	0	0	0	0	9	3	1	0	0	0		c_2	=	2.5
0	0	0	0	0	16	4	1	0	0	0		a_3		1
0	0	0	0	0	0	0	0	16	4	1		b_3		1
0	0	0	0	0	0	0	0	36	6	1		c_3		1
1	0	-2	-1	0	0	0	0	0	0	0		a_4		0
0	0	6	1	0	-6	-1	0	0	0	0		b_4		0
0	0	0	0	0	8	1	0	-8	-1	0		c_4		0

A Sol B

Solving in Matlab with sol = (A\B)

coeffecients = $b_1 = 1$ $c_1 = 1$ $a_2 = -.375$ $b_2 = 1.75$ $c_2 = 0.625$

$a_3 = -1$ $b_3 = 5.5$ $c_3 = -5$ $a_4 = 1.25$ $b_4 = -12.5$

$c_4 = 31$

$f_1 = x + 1$ $\quad f_2 = -.375x^2 + 1.75x + 0.625$

$f_3 = -x^2 + 5.5x - 5$ $\quad f_4 = 1.25x^2 - 12.5x + 31$

b) see matlab script 3, Fig 3

```
clc

a = [0:.01:1];
b = [1:.01:3];
c = [3:.01:4];
d = [4:.01:6];

f1 = a+1;
f2 = -.375.*b.^2+1.75.*b+.625;
f3 = -1.*c.^2+5.5.*c-5;
f4 = 1.25.*d.^2-12.5.*d+31;

x = [0 1 3 4 6];
y = [1 2 2.5 1 1];

hold on
plot(a,f1,b,f2,c,f3,d,f4)
scatter(x,y)
```

FIG 3