

Prob 1 Solution (Thanks to Gerald O'Neill)

1 The set of following 4 pts is given

x	y
-2	0
1	-1.95
3	-1.25
5	-1.75

a) using Lagrange interp method

Lagrange functions

$$L_1 = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} = \frac{(x-1)(x-3)(x-5)}{(-2-1)(-2-3)(-2-5)} = \frac{x^3 - 9x^2 + 23x - 15}{-105}$$

$$L_2 = \frac{(x+2)(x-3)(x-5)}{(1+2)(1-3)(1-5)} = \frac{x^3 - 6x^2 - x + 30}{24}$$

$$L_3 = \frac{(x+2)(x-1)(x-5)}{(3+2)(3-1)(3-5)} = \frac{x^3 - 4x^2 - 7x + 10}{-20}$$

$$L_4 = \frac{(x+2)(x-1)(x-3)}{(5+2)(5-1)(5-3)} = \frac{x^3 - 2x^2 - 5x + 6}{56}$$

$$\text{Poly} = \sum y_i L_i$$

$$y(x) = y_1 L_1 + y_2 L_2 + y_3 L_3 + y_4 L_4$$

$$y(x) = -0.05x^3 + 0.3x^2 - 0.2x - 2 \quad \text{calc at } x=2$$

$$\boxed{x=2 \quad y=-1.6}$$

b) i) plot pts, poly ii) four lagrange functions

i) SEE MATLAB SCRIPT 1, FIG 1

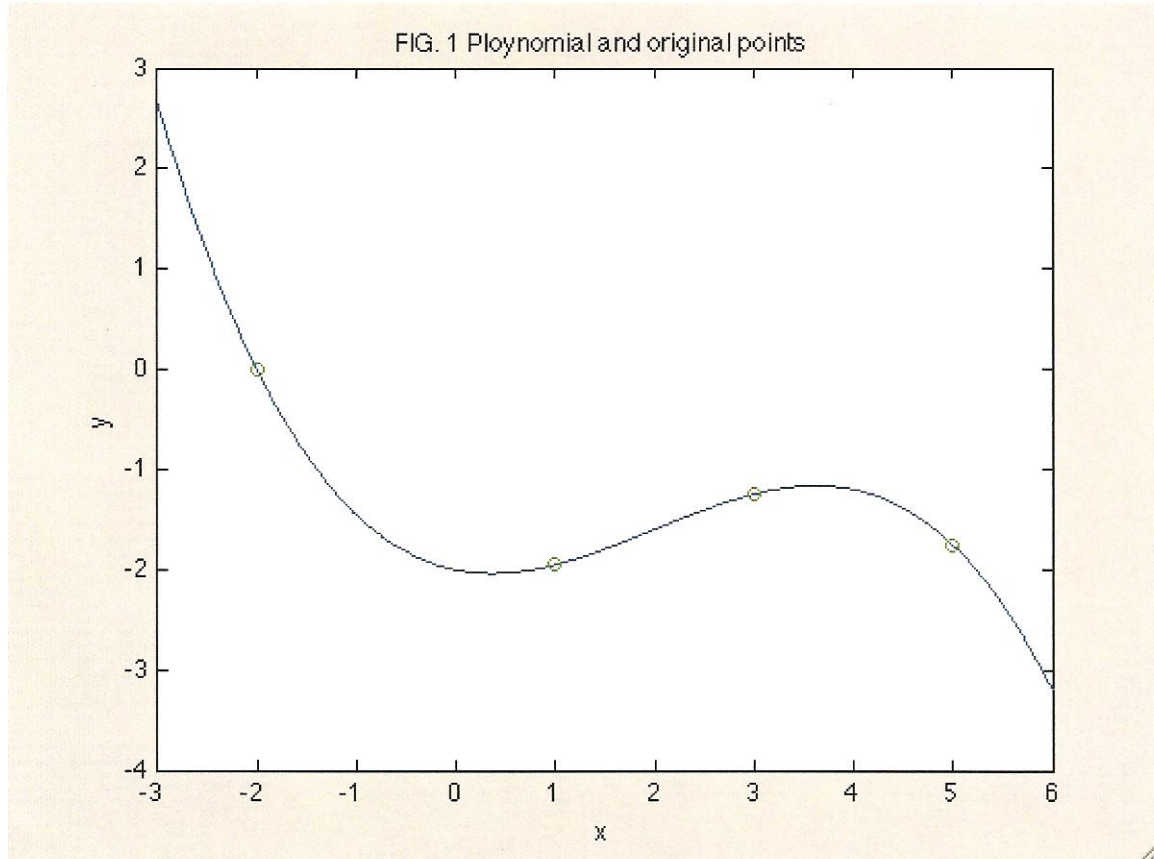
ii) SEE MATLAB SCRIPT 2, FIG 2

problem 2 on next page

```
clear all
clc

x = [-3:.01:6];
y = (-.05).*x.^3+(.3).*x.^2+(-.2).*x-2;
plot(x,y)
hold on
xo = [-2 1 3 5];
yo = [0 -1.95 -1.25 -1.75];
scatter(xo,yo)
```

FIG 1



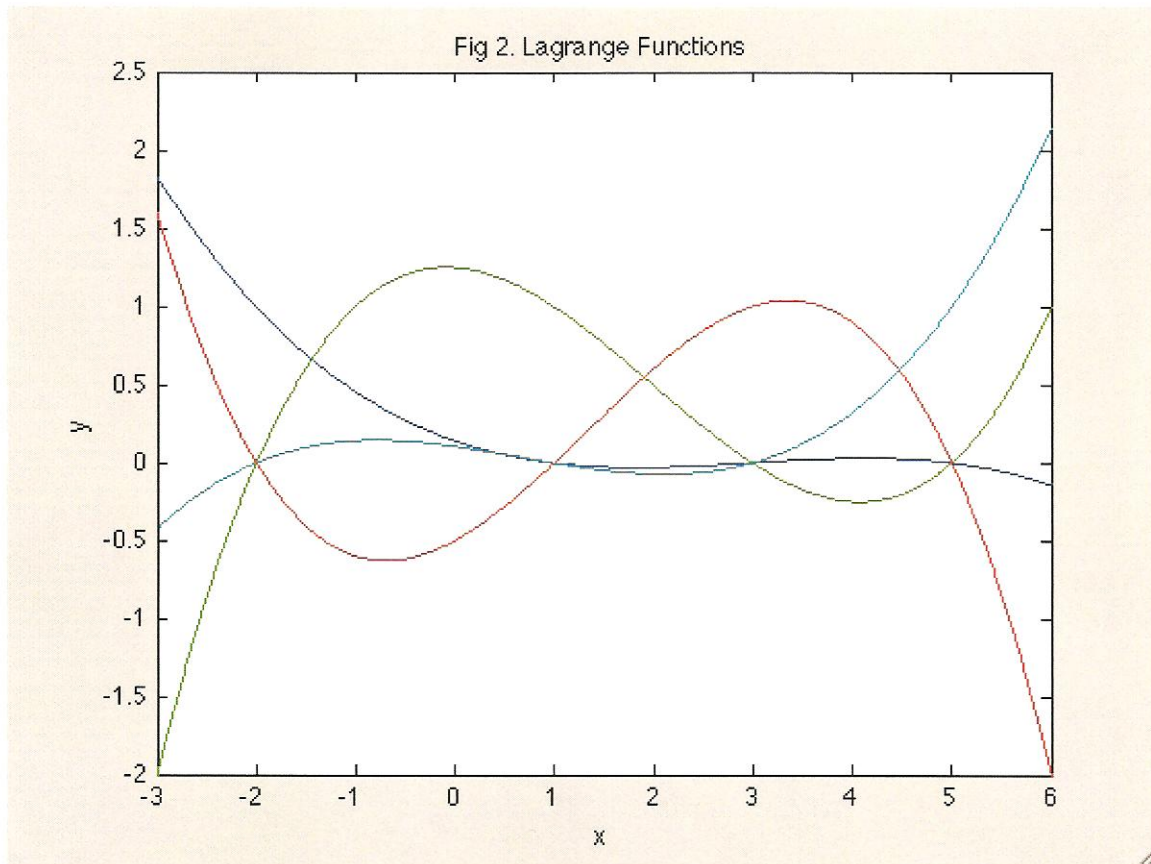
```

clear all
clc

hold off
x = [-3:.01:6];
L1 = (x.^3-9.*x.^2+23.*x-15)/-105;
L2 = (x.^3-6.*x.^2-1.*x+30)/24;
L3 = (x.^3-4.*x.^2-7.*x+10)/-20;
L4 = (x.^3-2.*x.^2-5.*x+6)/56;
plot(x,L1,x,L2,x,L3,x,L4)

```

FIG 2



Note by HPH: The 4 curves shown are the Lagrange functions, L_1 , L_2 , L_3 , and L_4 . Some students plotted y_1L_1 , y_2L_2 , y_3L_3 , and y_4L_4 , which are also acceptable. In that case, the structure of L_1 will be suppressed since $y_1 = 0$. Nevertheless, one should still plot y_1L_1 (a flat line) to complete the solution.

Prob 2 Solution (Thanks to Gerald O'Neill)

derivatives
 $2a_1x_2 + b_1 = 2a_2x_2 + b_2$
 $2a_2x_3 + b_2 = 2a_3x_3 + b_3$
 $2a_3x_4 + b_3 = 2a_4x_4 + b_4$

$b_1 - 2a_2 - b_2 = 0$
 $b_2 + 6a_2 - 6a_3 - b_3 = 0$
 $b_3 + 8a_3 - 8a_4 - b_4 = 0$

2 Set 5 pts given

x	y
0	1
1	2
3	2.5
4	1
6	1

$f_1 = b_1x_1 + c_1 = y_1$
 $b_1x_2 + c_1 = y_2$
 $f_2 = a_2x_2^2 + b_2x_2 + c_2 = y_2$
 $a_2x_3^2 + b_2x_3 + c_2 = y_3$
 $f_3 = a_3x_3^2 + b_3x_3 + c_3 = y_3$
 $a_3x_4^2 + b_3x_4 + c_3 = y_4$
 $f_4 = a_4x_4^2 + b_4x_4 + c_4 = y_4$
 $a_4x_5^2 + b_4x_5 + c_4 = y_5$

$0 \cdot b_1 + c_1 = 1$
 $1 \cdot b_1 + c_1 = 2$
 $1 \cdot a_2 + 1 \cdot b_2 + c_2 = 2$
 $9 \cdot a_2 + 3 \cdot b_2 + c_2 = 2.5$
 $9 \cdot a_3 + 3 \cdot b_3 + c_3 = 2.5$
 $16 \cdot a_3 + 4 \cdot b_3 + c_3 = 1$
 $16 \cdot a_4 + 4 \cdot b_4 + c_4 = 1$
 $36 \cdot a_4 + 6 \cdot b_4 + c_4 = 1$

b_1	c_1	a_2	b_2	c_2	a_3	b_3	c_3	a_4	b_4	c_4		
0	1	0	0	0	0	0	0	0	0	0	b_1	1
1	1	0	0	0	0	0	0	0	0	0	c_1	2
0	0	1	1	1	0	0	0	0	0	0	a_2	2
0	0	9	3	1	0	0	0	0	0	0	b_2	2.5
0	0	0	0	0	9	3	1	0	0	0	c_2	2.5
0	0	0	0	0	16	4	1	0	0	0	a_3	1
0	0	0	0	0	0	0	0	16	4	1	b_3	1
0	0	0	0	0	0	0	0	36	6	1	c_3	1
1	0	-2	-1	0	0	0	0	0	0	0	a_4	0
0	0	6	1	0	-6	-1	0	0	0	0	b_4	0
0	0	0	0	0	8	1	-8	-1	0	0	c_4	0

A Sol B

Solving in Matlab with $\text{sol} = (A \setminus B)$
 coefficients = $b_1 = 1$ $c_1 = 1$ $a_2 = -3.75$ $b_2 = 1.75$ $c_2 = 0.625$
 $a_3 = -1$ $b_3 = 5.5$ $c_3 = -5$ $a_4 = 1.25$ $b_4 = -12.5$
 $c_4 = 31$

$f_1 = x + 1$ $f_2 = -3.75x^2 + 1.75x + 0.625$
 $f_3 = -x^2 + 5.5x - 5$ $f_4 = 1.25x^2 - 12.5x + 31$

b) see matlab script 3, Fig 3

```

clc

a = [0:.01:1];
b = [1:.01:3];
c = [3:.01:4];
d = [4:.01:6];

f1 = a+1;
f2 = -.375.*b.^2+1.75.*b+.625;
f3 = -1.*c.^2+5.5.*c-5;
f4 = 1.25.*d.^2-12.5.*d+31;

x = [0 1 3 4 6];
y = [1 2 2.5 1 1];

hold on
plot(a,f1,b,f2,c,f3,d,f4)
scatter(x,y)

```

FIG 3

