MAE384 Fall 2011 Homework #4

In all problems, the argument of a sinusoidal function is always in radian.

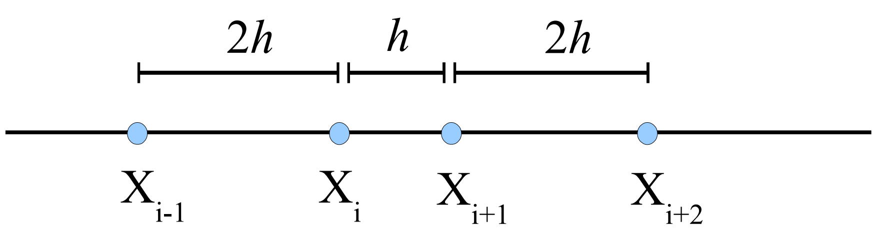
1. Evaluate the first derivative of the function, $f(x) = \sin(\exp(x))$, for the interval $0 \le x \le 4$. First, find $f'(x)$ analytically to prepare for later discussions. **(a)** Evaluate $f'(x)$ at the discrete points of $x = 0, 0.1, 0.2, ..., 3.9, 4.0$ by setting $h = 0.1$ and using the following two formulas: (i) The 2-point central difference scheme (3rd formula from top in p. 260), and (ii) The 4-point central difference scheme (4th formula from top in p. 260). Plot the numerical results and analytic solution (total of 3 curves) in a single figure. **(b)** Repeat (a) but now set $h = 0.01$ and evaluate $f'(x)$ at $x = 0, 0.01, 0.02, ..., 3.99, 4.0$. **(c)** Discuss the results in (a) and (b). In particular, you will notice that the performance of the finite difference scheme is not uniform in x. For our problem, the magnitude of numerical error generally increases with an increasing x. Explain why that's the case. **[4 points]**

(Note: It is understood that you will not be able to evaluate $f'(0)$ and $f'(4)$ using the 2-point scheme, and $f'(x)$ at the two leftmost and two rightmost points using the 4-point scheme. In those cases, for graphic purposes you may simply use the $f'(x)$ from the neighboring points to fill the missing values. For example, in Part (a) with the 2-point scheme you may set $f'(0) = f'(0.1)$, $f'(4) = f'(3.9)$, and so on.)

2. Consider the non-uniform grid (shown in the diagram below) with $x_i - x_{i-1} = 2h$, $x_{i+1} - x_i = h$, and $x_{i+2} - x_{i+1} = 2h$. Derive a 4-point finite difference formula for the <u>second derivative</u> of $f(x)$ that has a truncation error of $O(h^2)$. Your formula should have the form:

$$f''(x_i) = A\,f(x_{i-1}) + B\,f(x_i) + C\,f(x_{i+1}) + D\,f(x_{i+2}) + O(h^2) \quad .$$

Please clearly describe what your A, B, C, and D are in the final answer. **[2.5 points]**



3. All of the formula in Table 6-1 have a truncation error of $O(h)$, $O(h^2)$, or $O(h^4)$. Try to derive a six-point finite difference formula for the <u>first derivative</u> of $f(x)$ that has a truncation error of $O(h^5)$. Moreover, the formula must have the following form:

$$f'(x_i) = A\,f(x_{i-2}) + B\,f(x_{i-1}) + C\,f(x_i) + D\,f(x_{i+1}) + E\,f(x_{i+2}) + F\,f(x_{i+3}) + O(h^5) \quad .$$

In other words, the six points should include x_i itself, two points to its left and three points to its right. The spacing between two adjacent grid points is h = constant. After solving the problem, write specifically what your A, B, C, D, E, and F are. **[3.5 points]**