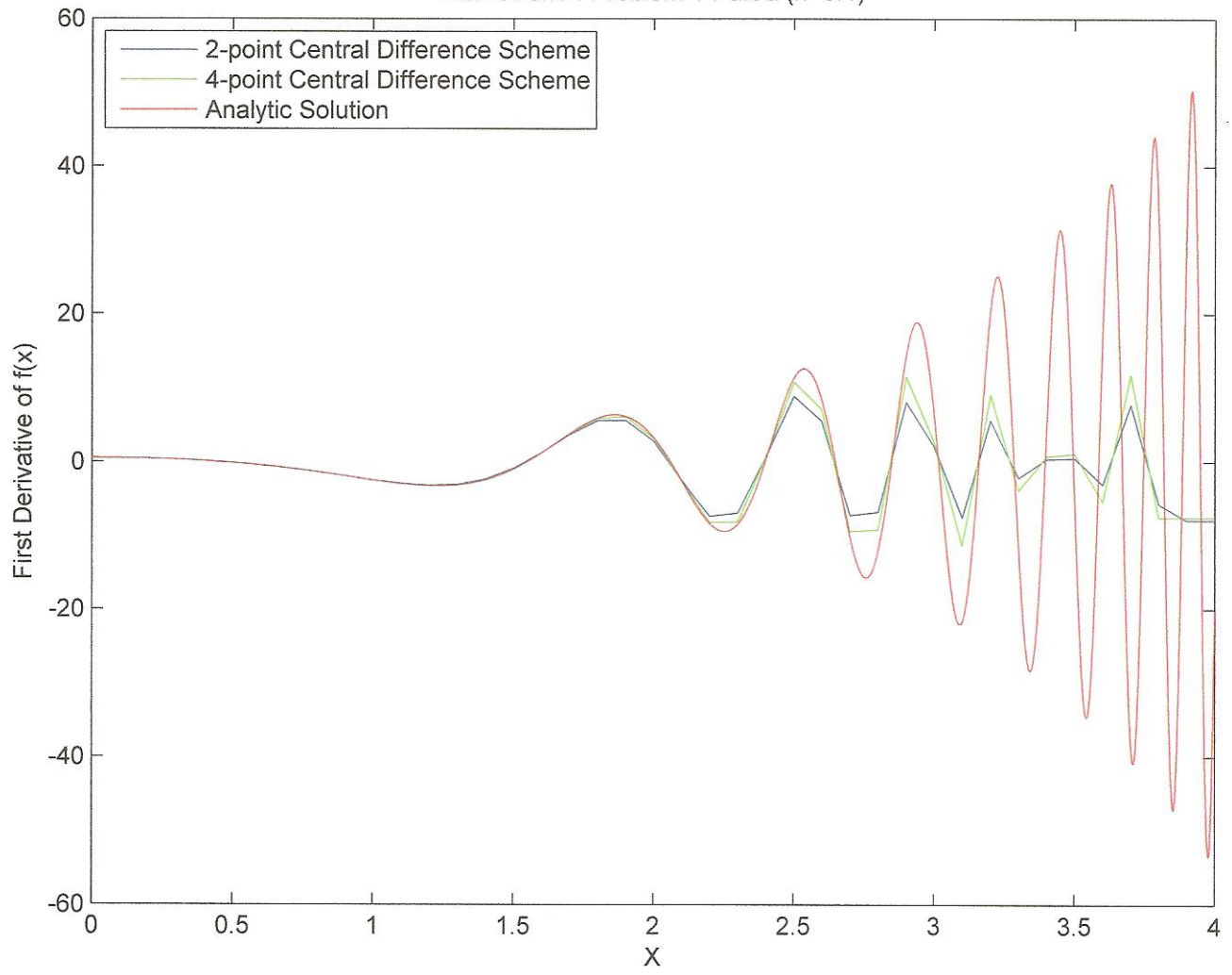


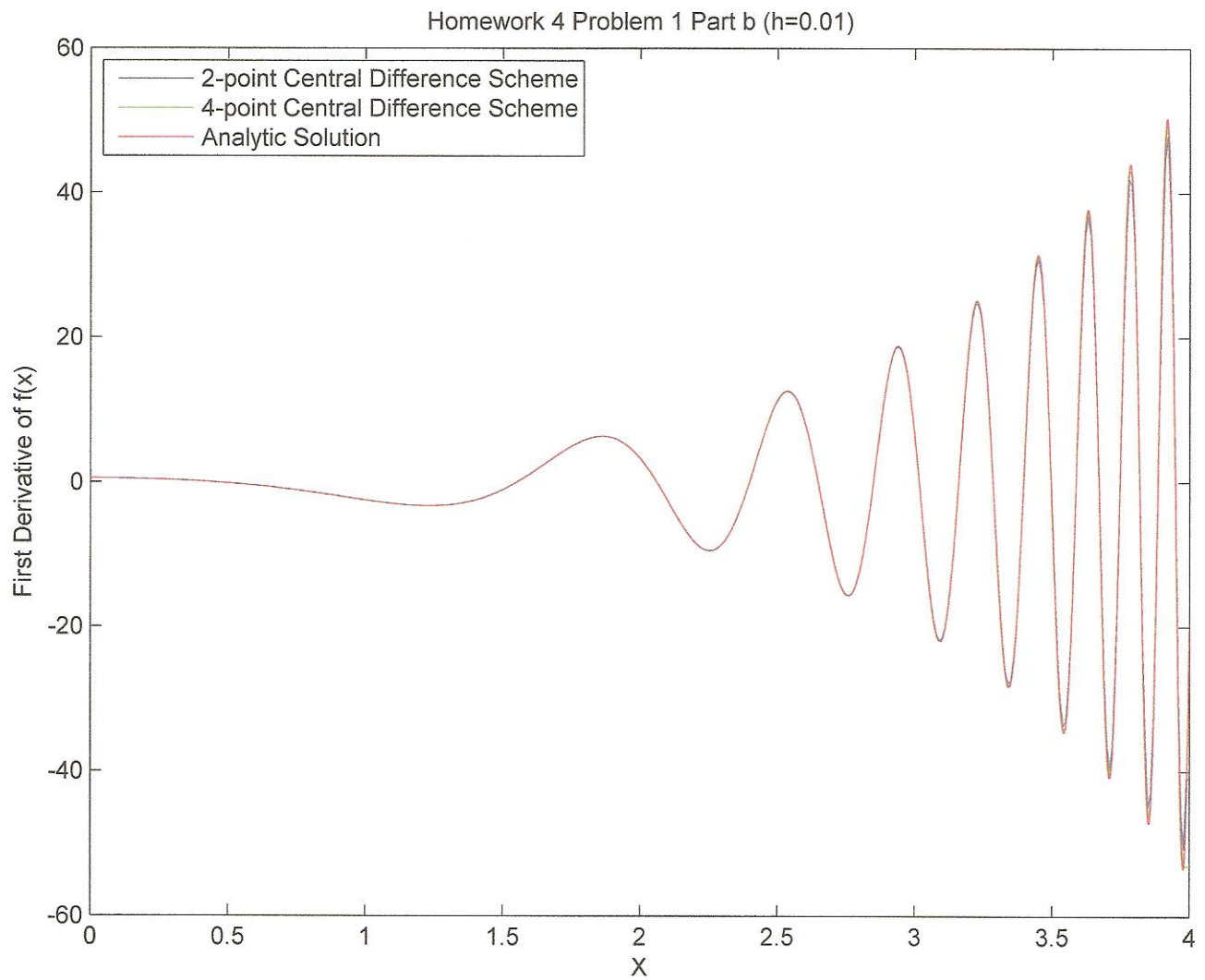
## Prob 1 Solution (Thanks to Kevin Antrosiglio)

```
% HOMEWORK #4 PROBLEM #1
x1 = 0:0.1:4;
h1 = 0.1;
for z1 = 2:40
    central2(z1) = (sin(exp(x1(z1+1))) - sin(exp(x1(z1-1))))/(2*h1);
end
central2(1) = central2(2);
central2(41) = central2(40);
plot(x1,central2)
hold on
for z2 = 3:39
    central4(z2) = (sin(exp(x1(z2-2))) - 8*sin(exp(x1(z2-1))) + 8*sin(exp(x1(z2+1))) -
sin(exp(x1(z2+2))))/(12*h1);
end
central4(1) = central4(3);
central4(2) = central4(3);
central4(40) = central4(39);
central4(41) = central4(39);
plot(x1,central4,'g')
x = 0:0.0001:4;
fprime = exp(x).*cos(exp(x));
plot(x,fprime,'r')
legend('2-point Central Difference Scheme','4-point Central Difference Scheme','Analytic
Solution','Location','NorthWest')
xlabel('X')
ylabel('First Derivative of f(x)')
title('Homework 4 Problem 1 Part a (h=0.1)')
x2 = 0:0.01:4;
h2 = 0.01;
for z3 = 2:400
    central_2(z3) = (sin(exp(x2(z3+1))) - sin(exp(x2(z3-1))))/(2*h2);
end
central_2(1) = central_2(2);
central_2(401) = central_2(400);
figure
plot(x2,central_2)
hold on
for z4 = 3:399
    central_4(z4) = (sin(exp(x2(z4-2))) - 8*sin(exp(x2(z4-1))) + 8*sin(exp(x2(z4+1))) -
sin(exp(x2(z4+2))))/(12*h2);
end
central_4(1) = central_4(3);
central_4(2) = central_4(3);
central_4(400) = central_4(399);
central_4(401) = central_4(399);
plot(x2,central_4,'g')
plot(x,fprime,'r')
legend('2-point Central Difference Scheme','4-point Central Difference Scheme','Analytic
Solution','Location','NorthWest')
xlabel('X')
ylabel('First Derivative of f(x)')
```

**Plots in next 2 pages**

Homework 4 Problem 1 Part a (h=0.1)





Note: For  $h = 0.01$  the numerical solutions are almost indistinguishable from the analytic solution.

Prob 2 Solution (Thanks to Olen Hatch)

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2} + \frac{f'''(x_i)h^3}{6} + O(h^4)$$

$$f(x_{i+2}) = f(x_i) + f'(x_i)(3h) + \frac{f''(x_i)(3h)^2}{2} + \frac{f'''(x_i)(3h)^3}{6} + O(h^4)$$

$$f(x_{i-1}) = f(x_i) + f'(x_i)(-2h) + \frac{f''(x_i)(-2h)^2}{2} + \frac{f'''(x_i)(-2h)^3}{6} + O(h^4)$$

a (  $f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{1}{2}f''(x_i)h^2 + \frac{1}{6}f'''(x_i)h^3 + O(h^4)$  )

b (  $f(x_{i+2}) = f(x_i) + 3f'(x_i)h + \frac{9}{2}f''(x_i)h^2 + \frac{27}{6}f'''(x_i)h^3 + O(h^4)$  )

1 (  $f(x_{i-1}) = f(x_i) - 2f'(x_i)h + 2f''(x_i)h^2 - \frac{8}{6}f'''(x_i)h^3 + O(h^4)$  )

$$\begin{aligned} a + 3b - 2 &= 0 \\ \frac{1}{6}a + \frac{27}{6}b - \frac{8}{6} &= 0 \end{aligned} \quad \left[ \begin{array}{cc|c} 1 & 3 & 2 \\ \frac{1}{6} & \frac{27}{6} & \frac{8}{6} \end{array} \right] \xrightarrow[\text{PIV3}]{\text{RREF}} \left[ \begin{array}{cc|c} 1 & 0 & 5/4 \\ 0 & 1 & 1/4 \end{array} \right]$$

$$a = 5/4 \quad b = 1/4$$

$$\frac{5}{4}f(x_{i+1}) = \frac{5}{4}f(x_i) + \frac{5}{4}f'(x_i)h + \frac{5}{8}f''(x_i)h^2 + \frac{5}{24}f'''(x_i)h^3 + O(h^4)$$

$$\frac{1}{4}f(x_{i+2}) = \frac{1}{4}f(x_i) + \frac{3}{4}f'(x_i)h + \frac{9}{8}f''(x_i)h^2 + \frac{27}{24}f'''(x_i)h^3 + O(h^4)$$

$$f(x_{i-1}) = f(x_i) - 2f'(x_i)h + 2f''(x_i)h^2 - \frac{8}{6}f'''(x_i)h^3 + O(h^4)$$

$$f(x_{i-1}) + \frac{5}{4}f(x_{i+1}) + \frac{1}{4}f(x_{i+2}) = \frac{5}{2}f(x_i) + \frac{15}{4}f'(x_i)h^2 + O(h^4)$$

$$\frac{4}{15h^2} \left[ f(x_{i-1}) - \frac{5}{2}f(x_i) + \frac{5}{4}f(x_{i+1}) + \frac{1}{4}f(x_{i+2}) = \frac{15h^2}{4}f''(x_i) + O(h^4) \right]$$

$$f''(x_i) = \frac{4}{15h^2}f(x_{i-1}) - \frac{2}{3h^2}f(x_i) + \frac{1}{3h^2}f(x_{i+1}) + \frac{1}{15h^2}f(x_{i+2}) + O(h^2)$$

thus

$$A = \frac{4}{15h^2}$$

$$B = \frac{-2}{3h^2}$$

$$C = \frac{1}{3h^2}$$

$$D = \frac{1}{15h^2}$$

Prob 3 Solution (Thanks to Olen Hatch)

$$a \left[ f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2} + \frac{f'''(x_i)h^3}{6} + \frac{f^{IV}(x_i)h^4}{24} + \frac{f^V(x_i)h^5}{120} + O(h^6) \right]$$

$$b \left[ f(x_{i+2}) = f(x_i) + f'(x_i)(2h) + \frac{f''(x_i)(2h)^2}{2} + \frac{f'''(x_i)(2h)^3}{6} + \frac{f^{IV}(x_i)(2h)^4}{24} + \frac{f^V(x_i)(2h)^5}{120} + O(h^6) \right]$$

$$c \left[ f(x_{i+3}) = f(x_i) + f'(x_i)(3h) + \frac{f''(x_i)(3h)^2}{2} + \frac{f'''(x_i)(3h)^3}{6} + \frac{f^{IV}(x_i)(3h)^4}{24} + \frac{f^V(x_i)(3h)^5}{120} + O(h^6) \right]$$

$$d \left[ f(x_{i-1}) = f(x_i) + f'(x_i)(-h) + \frac{f''(x_i)(-h)^2}{2} + \frac{f'''(x_i)(-h)^3}{6} + \frac{f^{IV}(x_i)(-h)^4}{24} + \frac{f^V(x_i)(-h)^5}{120} + O(h^6) \right]$$

$$1 \left[ f(x_{i-2}) = f(x_i) + f'(x_i)(-2h) + \frac{f''(x_i)(-2h)^2}{2} + \frac{f'''(x_i)(-2h)^3}{6} + \frac{f^{IV}(x_i)(-2h)^4}{24} + \frac{f^V(x_i)(-2h)^5}{120} + O(h^6) \right]$$

$$\frac{a}{2} + \frac{4b}{2} + \frac{9c}{2} + \frac{d}{2} + \frac{4}{2} = 0$$

$$\frac{a}{6} + \frac{8b}{6} + \frac{27c}{6} - \frac{d}{6} - \frac{8}{6} = 0$$

$$\frac{a}{24} + \frac{16b}{24} + \frac{81c}{24} + \frac{d}{24} + \frac{16}{24} = 0$$

$$\frac{a}{120} + \frac{32b}{120} + \frac{243c}{120} - \frac{d}{120} - \frac{32}{120} = 0$$

$$\left[ \begin{array}{cccc|c} 1/2 & 2 & 9/2 & 1/2 & -2 \\ 1/6 & 4/3 & 9/2 & -1/6 & 4/3 \\ 1/24 & 16/24 & 81/24 & 1/24 & -16/24 \\ 1/120 & 32/120 & 243/120 & -1/120 & 32/120 \end{array} \right] \xrightarrow{\text{RREF}} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 2/3 \\ 0 & 0 & 0 & 1 & -10 \end{array} \right]$$

$$a = 20 \quad b = -5 \quad c = \frac{2}{3} \quad d = -10$$

Continue to next page

Prob 3 continued

$$20 \left[ f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{1}{2}f''(x_i)h^2 + \frac{1}{6}f'''(x_i)h^3 + \frac{1}{24}f^{(4)}(x_i)h^4 + \frac{1}{120}f^{(5)}(x_i)h^5 + O(h^6) \right]$$

$$-5 \left[ f(x_{i+2}) = f(x_i) + 2f'(x_i)h + 2f''(x_i)h^2 + \frac{8}{6}f'''(x_i)h^3 + \frac{16}{24}f^{(4)}(x_i)h^4 + \frac{32}{120}f^{(5)}(x_i)h^5 + O(h^6) \right]$$

$$\frac{2}{3} \left[ f(x_{i+3}) = f(x_i) + 3f'(x_i)h + \frac{9}{2}f''(x_i)h^2 + \frac{27}{6}f'''(x_i)h^3 + \frac{81}{24}f^{(4)}(x_i)h^4 + \frac{243}{120}f^{(5)}(x_i)h^5 + O(h^6) \right]$$

$$-10 \left[ f(x_{i-1}) = f(x_i) - f'(x_i)h + \frac{1}{2}f''(x_i)h^2 - \frac{1}{6}f'''(x_i)h^3 + \frac{1}{24}f^{(4)}(x_i)h^4 - \frac{1}{120}f^{(5)}(x_i)h^5 + O(h^6) \right]$$

$$1 \left[ f(x_{i-2}) = f(x_i) - 2f'(x_i)h + \frac{4}{2}f''(x_i)h^2 - \frac{8}{6}f'''(x_i)h^3 + \frac{16}{24}f^{(4)}(x_i)h^4 - \frac{32}{120}f^{(5)}(x_i)h^5 + O(h^6) \right]$$

$$20f(x_{i+1}) = 20f(x_i) + 20f'(x_i)h + 10f''(x_i)h^2 + \frac{20}{6}f'''(x_i)h^3 + \frac{20}{24}f^{(4)}(x_i)h^4 + \frac{20}{120}f^{(5)}(x_i)h^5 + O(h^6)$$

$$-5f(x_{i+2}) = -5f(x_i) - 10f'(x_i)h - 10f''(x_i)h^2 - \frac{40}{6}f'''(x_i)h^3 - \frac{80}{24}f^{(4)}(x_i)h^4 - \frac{160}{120}f^{(5)}(x_i)h^5 + O(h^6)$$

$$\frac{2}{3}f(x_{i+3}) = \frac{2}{3}f(x_i) + 2f'(x_i)h + 3f''(x_i)h^2 + 3f'''(x_i)h^3 + \frac{9}{4}f^{(4)}(x_i)h^4 + \frac{27}{20}f^{(5)}(x_i)h^5 + O(h^6)$$

$$-10f(x_{i-1}) = -10f(x_i) + 10f'(x_i)h - 5f''(x_i)h^2 + \frac{10}{6}f'''(x_i)h^3 - \frac{10}{24}f^{(4)}(x_i)h^4 + \frac{10}{120}f^{(5)}(x_i)h^5 + O(h^6)$$

$$f(x_{i-2}) = f(x_i) - 2f'(x_i)h + 2f''(x_i)h^2 - \frac{8}{6}f'''(x_i)h^3 + \frac{16}{24}f^{(4)}(x_i)h^4 - \frac{32}{120}f^{(5)}(x_i)h^5 + O(h^6)$$

$$f(x_{i-2}) - 10f(x_{i-1}) + 20f(x_{i+1}) - 5f(x_{i+2}) + \frac{2}{3}f(x_{i+3}) = \frac{20}{3}f(x_i) + 20f'(x_i)h + O(h^6)$$

$$\frac{1}{20h} \left( f(x_{i-2}) - 10f(x_{i-1}) - \frac{20}{3}f(x_i) + 20f(x_{i+1}) - 5f(x_{i+2}) + \frac{2}{3}f(x_{i+3}) \right) + O(h^5) = 20f'(x_i)h$$

$$f'(x_i) = \frac{1}{20h} f(x_{i-2}) - \frac{1}{2h} f(x_{i-1}) - \frac{1}{3h} f(x_i) + \frac{f(x_{i+1})}{h} - \frac{1}{4h} f(x_{i+2}) + \frac{1}{30h} f(x_{i+3}) + O(h^5)$$

so  $A = \frac{1}{20h}$     $B = -\frac{1}{2h}$     $C = -\frac{1}{3h}$     $D = \frac{1}{h}$

$E = -\frac{1}{4h}$     $F = \frac{1}{30h}$

