MAE384 Fall 2011 HW5

In all problems, the argument of a sinusoidal function is always in radian.
For Prob 1, 3, and 4, please submit the computer codes that produce your numerical solutions.

1. Evaluate the following integral,

$$I = \int_0^6 e^x \cos(e^x)\, dx \;,$$

using the composite Simpson's 3/8 method for the three cases with $h = 0.1$, 0.01, and 0.001. Compare the results with the analytic solution. **[2 points]**

2. An integral is defined by

$$I = \int_0^4 (3x^4 + 5x^3 - 7x + 3)\, dx \;.$$

First, evaluate it analytically. Then, evaluate it using Gauss quadrature by the following procedure:
(i) Transform the integral to the standard form, $I = \int_{-1}^{1} f(X)\, dX$, where $X = x/2 - 1$. (ii) Use the 3-point Gauss quadrature in Table 7-1 to perform the numerical integration. Compare the result with the analytic solution. **[0.5 point]**

3. (a) Solve the following initial value problem,

$$\frac{du}{dx} = x\, e^{-u} - 1 \;, u(0) = 0 \,,$$

using the classical 3rd order Runge-Kutta method with $h = 0.1$. Find the solution $u(x)$ for the interval of $0 \le x \le 4$. Also, solve it analytically, then compare the numerical and analytic solutions by plotting them together. Beware that the first term in the right hand side is $x \exp(-u)$, not $x \exp(-x)$. **[2.5 points]**

(b) Same as **(a)** but use the Euler's implicit method with $h = 0.1$ to find the numerical solution for $0 \le x \le 4$. Compare the numerical and analytic solutions by plotting them in a single figure. **[2.5 points]**

4. Solve the initial value problem,

$$\frac{d^2u}{dx^2} + 2\frac{du}{dx} - 3u = 0 \;,$$

with the initial conditions: (I) $u(0) = 3$, (II) $u'(0) = -1$ (u' is du/dx),

using the following methods: (i) Use the 3-point central difference scheme (9th formula from top in p. 260) to represent u'' in the ODE. (ii) Use the 2-point forward difference scheme (1st formula in Table 6-1 in p. 259) to represent u' in the ODE and the 2nd initial condition. Find the numerical solution for the interval, $0 \le x \le 2$, for the two cases: $h = 0.1$ and $h = 0.05$. (In each case, use the same h in (i) and (ii)). Find the analytic solution and compare the numerical and analytic solutions. **[2.5 points]**