Prob 1 Solution (Thanks to Kenneth Frazier)

```
clc
clear
r=quad('exp(x).*cos(exp(x))',0,6); %analytic solution
fprintf('    solution for problem 1 using simpson 3/8')
fprintf('\r       h          I              actual       relative error \r')
for h=[.1,.01,.001]
    h1=1*h;
    h2=2*h;
    h3=3*h;
    a=0;b=6;
    N=(b-a)/h;
    fa=exp(a).*cos(exp(a));
    fb=exp(b).*cos(exp(b));
    t1=0;
for i=1:3:(N-1)
    x1=h1;
    h1=x1+3*h;
    f1=(exp(x1).*cos(exp(x1)));
    t1=t1+f1;
end
t2=0;
for i=2:3:(N-1)
    x2=h2;
    h2=x2+3*h;
    f2=(exp(x2).*cos(exp(x2)));
    t2=t2+f2;
end
t3=0;
for i=3:3:((N-2))
    x3=h3;
    h3=x3+3*h;
    f3=(exp(x3).*cos(exp(x3)));
    t3=t3+f3;
end
I=(3/8)*h*(fa + 3*(t1) + 3*(t2) + 2*(t3) + fb);
e=((r-I)/r);
fprintf('\r %9.6f     %9.6f      %9.6f     %9.6f \r',h ,  I, r ,e)
end
```

Results:

```
solution for problem 1 using Simpson 3/8

      h          I              actual       relative error

 0.100000     40.049052       0.123396      -323.556813

 0.010000      1.254602       0.123396      -9.167273

 0.001000      0.123726       0.123396      -0.002675
```

Note: The analytic solution is $I = \sin(\exp(6)) - \sin(1) = 0.12339$ ...

Prob 2 Solution (Thanks to Joe Conlin)

$$I = \int_0^4 (3x^4 + 5x^3 - 7x + 3)\,dx$$

$$= \left[\tfrac{3}{5}x^5 + \tfrac{5}{4}x^4 - \tfrac{7}{2}x^2 + 3x \right|_0^4$$

$$= \tfrac{3}{5}(4)^5 + \tfrac{5}{4}(4)^4 - \tfrac{7}{2}(4)^2 + 3(4) = \boxed{890.4}$$ ✓

(i) standard form ⇒ $I = \int_{-1}^{1} f(X)\,dX$, where $X = x/2 - 1$

$\rightarrow x = 2X + 2$

$dx = 2\,dX$

$$\Rightarrow I = \int_{-1}^{1} \underbrace{[3(2X+2)^4 + 5(2X+2)^3 - 7(2X+2) + 3](2)}_{f(X)}\,dX$$

⇒ Table 7-1

$\rightarrow I \approx C_1 f(X_1) + C_2 f(X_2) + C_3 f(X_3)$

$C_1 = 0.555556$ $X_1 = -0.77459667$

$C_2 = 0.888889$ $X_2 = 0$

$C_3 = 0.5555556$ $X_3 = 0.77459667$

$$\Rightarrow \boxed{I \approx 890.40063208}$$ ✓

$$\Rightarrow \boxed{7.098 \times 10^{-6}\ \%\ \text{error}}$$

Prob 3 Solution - next 3 pages (Thanks to Joe Conlin)

Analytic solution:

$$\frac{du}{dx} = xe^{-u} - 1 \quad , \quad u(0) = 0$$

$$\Rightarrow \frac{du}{dx} + 1 = xe^{-u}$$

$\rightarrow u^* = u + x$

$\rightarrow xe^{-u} = f(x)\, e^{-(u+x)}$

$f(x) = \frac{xe^{-u}}{e^{-(u+x)}} = \frac{x}{e^{-x}} = xe^{x}$

$$\Rightarrow \frac{du^*}{dx} = xe^{x} e^{-u^*}$$

$$\Rightarrow \int \frac{du^*}{e^{-u^*}} = \int xe^{x} dx \Rightarrow \int e^{u^*} du^* = \int xe^{x} dx$$

$\rightarrow u = x \quad du = 1\, dx$

$dv = e^{x} dx \quad v = e^{x}$

$\int xe^{x} dx = xe^{x} - \int e^{x} dx = xe^{x} - e^{x}$

$$\Rightarrow \ln(e^{u^*}) = \ln(xe^{x} - e^{x} + c)$$

$$u^* = \ln(xe^{x} - e^{x} + c)$$

$$u = \ln(xe^{x} - e^{x} + c) - x$$

$\rightarrow$ i.c. $\Rightarrow u(0) = \ln(-0e^{0} - e^{0} + c) - 0 = 0$

$c = 2$

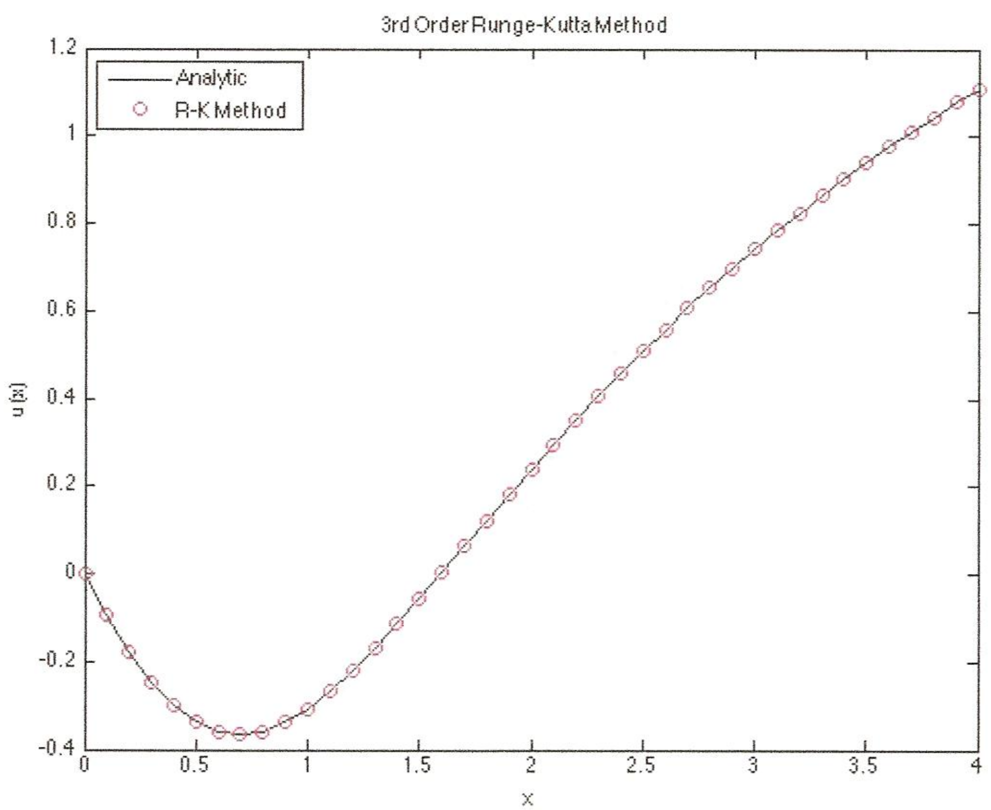
$$\boxed{u(x) = \ln\left[e^{x}(x-1) + 2\right] - x}$$

Prob 3(a) numerical solution

Problem 3

Part (a)

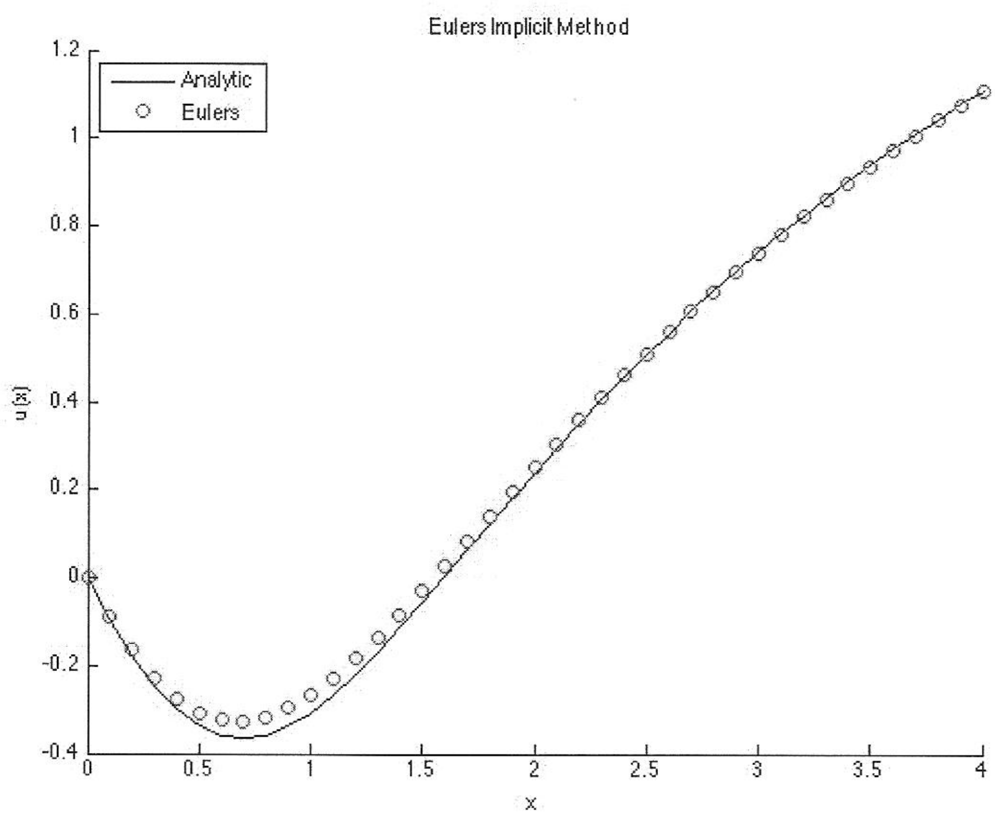
```
f = inline('x*exp(-u)-1','x','u');
xi = 0:.1:4;
ui = log(exp(xi).*(xi-1)+2)-xi;
plot(xi,ui,'k')
hold on
u = 0;
x = 0;
plot(u,x,'ro')
h = 0.1;
for i = 1:h:5-h
    k1 = f(x,u);
    k2 = f(x+h/2,u+k1*h/2);
    k3 = f(x+h,u-k1*h+2*k2*h);
    u = u+(1/6)*(k1+4*k2+k3)*h;
    x = x+h;
    plot(x,u,'ro')
end
axis([0 4 -.4 1.2])
title('3rd Order Runge-Kutta Method')
xlabel('x')
ylabel('u(x)')
legend('Analytic','R-K Method',2)
```



Prob 3(b) numerical solution

Problem 3
Part (b)
MATLAB Code:

```
hold on
f = inline('x*exp(-u)-1','x','u');
xi = 0:.1:4;
ui = log(exp(xi).*(xi-1)+2)-xi;
plot(xi,ui,'k')
u0 = 0;
x0 = 0;
plot(u0,x0,'ro')
h = 0.1;
for i = 1:1:40
    x = h*i;
    u = u0-(u0-u0+(1-x*exp(-u0))*h)/(1+x*exp(-u0)*h);
    u0 = u;
    plot(x,u,'ro')
end
title('Eulers Implicit Method')
xlabel('x')
ylabel('u(x)')
legend('Analytic','Eulers',2)
```



Note: The above matlab code used Newton's method to find the solution of the nonlinear equation at every step. There are other methods (fixed point iteration, etc.) that may also work. The accuracy of the solution would somewhat depend on the detail of the iterative process. Grading for this problem was based on both the accuracy of the solution and the correctness of the procedure.

Prob 4 Solution (Thanks to Taylin Dean)

The analytic solution is $u(x) = \exp(-3x) + 2\exp(x)$. The finite difference formula to use is

$$\frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} + \frac{2u_{i+1} - 2u_i}{h} - 3u_i = 0 \quad , \; i = 1, 2, ..., \mathrm{N} \, ,$$

with the initial conditions $u_0 = 3$ and $u_1 = u_0 - h$.

```
clear all
close all
clc
hold on

a = 0;
b = 2;
h = 0.1;
for j = 1:2
    x = a:h:b;
    N = (b-a)/h;

    U(1) = 3;
    U(2) = U(1)-h;

    A = exp(-3.*x)+2.*exp(x);

    for i = 2:N
        Num = ((3*h^2)+(2*h)+2).*U(i)-U(i-1);
        Den = 1+(2*h);
        U(i+1)= Num./Den;
    end
    if j == 1
       plot(x,A)
       plot(x,U,'r--')
       h = h/2;
    end
end
plot(x,U,'k.')
```