MAE384 Fall 2011 Homework #6

1. Solve the boundary value problem,

$$\frac{d^2 u}{d x^2} + x^2 u = 0 \quad ,$$

$$u(0) = 1 \ , \quad u'(5) = 0.5 \quad ,$$

for $u(x)$ within the domain of $0 \leq x \leq 5$. Use the 3-point central difference scheme (9th formula from top in p. 260) to represent u'' in the differential equation and 2-point backward finite difference scheme (1st formula in p. 260) to represent the u' in the second boundary condition. Choose $h = 0.1$. Plot your solution. **(4 points)**

2. **(a)** Solve the partial differential equation,

$$\frac{\partial u}{\partial t} = A \frac{\partial u}{\partial x} \quad ,$$

defined on the semi-infinite domain, $-\infty < x < \infty$ and $0 \leq t < \infty$, with the boundary condition given at $t = 0$ as

$$u(x, 0) = 1 \quad , \text{ if } 0 \leq x \leq 1$$
$$= 0 \quad , \text{ otherwise .}$$

Use the 2-point forward difference scheme (1st formula in Table 6-1 in p. 259) to represent $\partial u/\partial t$ and 2-point backward difference scheme (1st formula in p. 260) to represent $\partial u/\partial x$. Choose $A = -0.6$, $\Delta x = 0.1$ and $\Delta t = 0.1$. Integrate your system forward in t to find the solution, $u(x,t)$, at $t = 0.5$, 1, and 2. Plot these solutions (as a function of x) along with the "initial" state, $u(x,0)$, for the domain of $0 \leq x \leq 5$. (Note: Your solution could be non-zero outside this domain. In that case, you only need to plot the result for $0 \leq x \leq 5$.)
(b) Using the same finite difference scheme and the same Δx and Δt in (a), explore the behavior of the numerical solutions for the cases with $A = -1.2$ and $A = 0.6$. You do not have to integrate the system to $t = 2$. Integrating it for a few time steps (for example, to $t = 0.5$) would suffice to reveal the behavior of those solutions. Explain why the behavior of the solutions for these two cases differ from the case with $A = -0.6$ in Part (a). **(3 points)**

3. Find the general solution of the following PDEs by the method of separation of variables. **(1 point)**

(a) $$\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} + u = 0$$

(b) $$y \frac{\partial u}{\partial x} + x^2 \frac{\partial u}{\partial y} = 0$$