Prob 1
The derivation of the finite difference formula was discussed in class. Solution:


Matlab code
$\mathrm{h}=0.1$;
$x=[0.1: 0.1: 4.9] ; x p l o t=[0: 0.1: 5] ;$
$A=\operatorname{zeros}(49,49)$;
$A(1,1)=\left(h^{\wedge} 2\right) *\left(x(1)^{\wedge} 2\right)-2$;
$A(1,2)=1$;
$A(49,48)=1$;
$A(49,49)=\left(h^{\wedge} 2\right)^{\star}\left(x(49)^{\wedge} 2\right)-1$;
for $m=2: 48$
for $n=1: 49$
$A(m, n)=0$;
if $(\mathrm{n}=(\mathrm{m}-1))$
$A(m, n)=1 ;$
elseif $(\mathrm{n}==\mathrm{m})$ $A(m, n)=\left(h^{\wedge} 2\right) *\left(x(m)^{\wedge} 2\right)-2 ;$
elseif $(\mathrm{n}==\mathrm{m}+1)$
$A(m, n)=1 ;$
end
end
end
b $=\operatorname{zeros}(49)$;
b(1) $=-1$;
$\mathrm{b}(49)=-0.5 * \mathrm{~h}$;
$u 1=\operatorname{inv}(A)^{*} b^{\prime}$;
$u(1)=1$;
$u(51)=u 1(49)+0.5 * h$;
for $k=2: 50$
$u(k)=u 1(k-1) ;$
end
plot(xplot,u);xlabel('x');ylabel('u(x)')

Prob 2
Part (a): The derivation of the finite difference formula was given in class. Solution:


Matlab code in next page

Matlab code for Prob 2(a)

```
dx = 0.1; dt = 0.1; A = -0.6;
x = [-0.1:0.1:5.1];
xx = [0:0.1:5];
for k = 1:52
    if (x(k) >= 0) && (x(k) <= 1)
        u(k) = 1;
    else
        u(k) = 0;
    end
end
for k = 2:52
    u00(k-1) = u(k);
end
for n = 1:5
    for k = 2:52
        u1(k) = (1+A*dt/dx)*u(k)-(A*dt/dx)*u(k-1);
    end
    u1(1) = 0;
    for k = 1:52
        u(k) = ul(k);
    end
end
for k = 2:52
    u05(k-1) = u(k);
end
for n = 1:5
    for k = 2:52
        u1(k) = (1+A*dt/dx)*u(k)-(A*dt/dx)*u(k-1);
    end
    u1(1) = 0;
    for k = 1:52
        u(k) = ul(k);
    end
end
for k = 2:52
    u10(k-1) = u(k);
end
for n = 1:10
    for k = 2:52
        u1(k) = (1+A*dt/dx)*u(k)-(A*dt/dx)*u(k-1);
    end
    u1(1) = 0;
    for k = 1:52
        u(k) = ul(k);
    end
end
for k = 2:52
    u20(k-1) = u(k);
end
plot(xx,u00,'k-',xx,u05,'r-',xx,u10,'b-', xx,u20,'m-')
axis([0 5 0 1.2])
legend('t = 0','t = 0.5','t = 1','t = 2')
xlabel('x');ylabel('u(x,t)')
```

Prob 2 Part (b)
Results after integrating the system for a few steps:
$\mathrm{A}=0.6$

$\mathrm{A}=-1.2$


The reasons that the numerical scheme does not work for these cases were discussed in class.

Prob 3

We have discussed the detail of the solutions in class.

