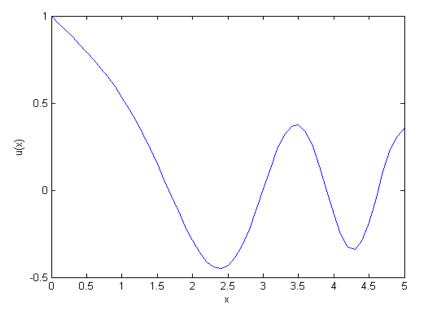
Prob 1

The derivation of the finite difference formula was discussed in class. Solution:

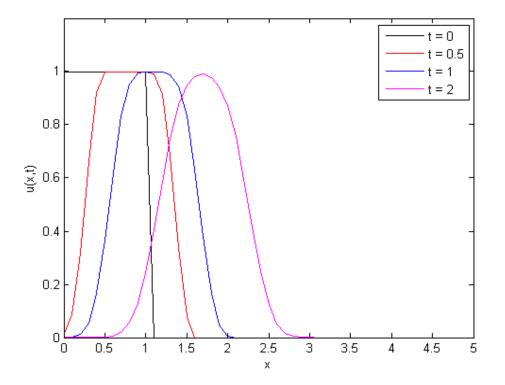


Matlab code

```
h = 0.1;
x = [0.1:0.1:4.9]; xplot = [0:0.1:5];
A = zeros(49, 49);
A(1,1) = (h^2) * (x(1)^2) - 2;
A(1,2) = 1;
A(49, 48) = 1;
A(49, 49) = (h^2) * (x(49)^2) - 1;
for m = 2:48
    for n = 1:49
        A(m, n) = 0;
         if (n == (m-1))
             A(m, n) = 1;
         elseif (n == m)
             A(m, n) = (h^2) * (x(m)^2) - 2;
         elseif (n == m+1)
             A(m, n) = 1;
         end
    end
end
b = zeros(49);
b(1) = -1;
b(49) = -0.5 * h;
u1 = inv(A) *b';
u(1) = 1;
u(51) = u1(49) + 0.5 * h;
for k = 2:50
    u(k) = u1(k-1);
end
plot(xplot, u);xlabel('x');ylabel('u(x)')
```

Prob 2

Part (a): The derivation of the finite difference formula was given in class. Solution:



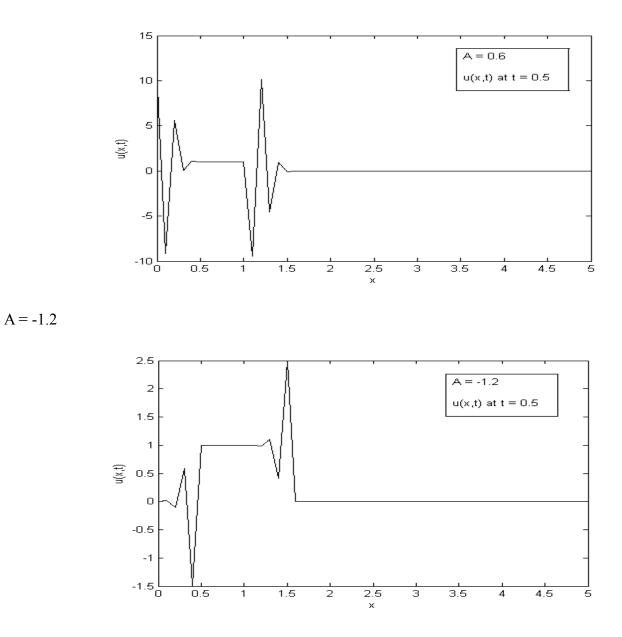
Matlab code in next page

Matlab code for Prob 2(a)

```
dx = 0.1; dt = 0.1; A = -0.6;
x = [-0.1:0.1:5.1];
xx = [0:0.1:5];
for k = 1:52
    if (x(k) \ge 0) \&\& (x(k) \le 1)
        u(k) = 1;
    else
        u(k) = 0;
    end
end
for k = 2:52
    u00(k-1) = u(k);
end
for n = 1:5
    for k = 2:52
        u1(k) = (1+A*dt/dx)*u(k) - (A*dt/dx)*u(k-1);
    end
    u1(1) = 0;
    for k = 1:52
        u(k) = u1(k);
    end
end
for k = 2:52
    u05(k-1) = u(k);
end
for n = 1:5
    for k = 2:52
        u1(k) = (1+A*dt/dx)*u(k) - (A*dt/dx)*u(k-1);
    end
    u1(1) = 0;
    for k = 1:52
        u(k) = u1(k);
    end
end
for k = 2:52
    u10(k-1) = u(k);
end
for n = 1:10
    for k = 2:52
        u1(k) = (1+A*dt/dx)*u(k) - (A*dt/dx)*u(k-1);
    end
    u1(1) = 0;
    for k = 1:52
        u(k) = u1(k);
    end
end
for k = 2:52
    u20(k-1) = u(k);
end
plot(xx,u00,'k-',xx,u05,'r-',xx,u10,'b-',xx,u20,'m-')
axis([0 5 0 1.2])
legend('t = 0', 't = 0.5', 't = 1', 't = 2')
xlabel('x');ylabel('u(x,t)')
```

Results after integrating the system for a few steps:





The reasons that the numerical scheme does not work for these cases were discussed in class.

Prob 3

We have discussed the detail of the solutions in class.