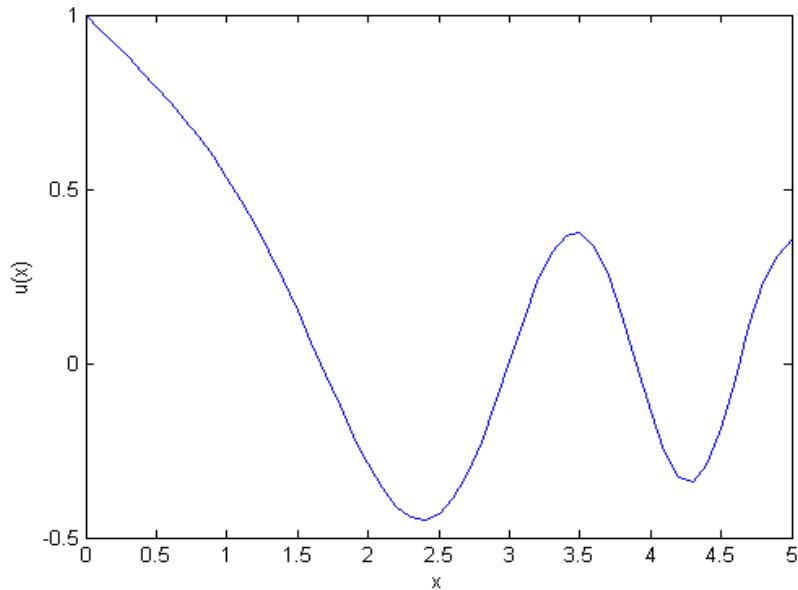


Prob 1

The derivation of the finite difference formula was discussed in class. Solution:

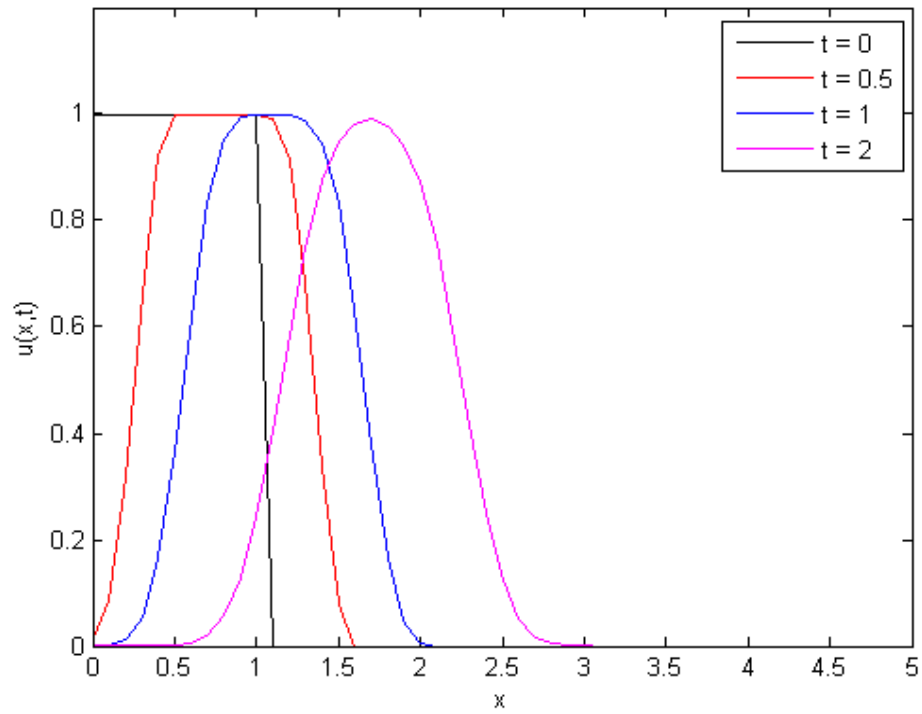


Matlab code

```
h = 0.1;
x = [0.1:0.1:4.9]; xplot = [0:0.1:5];
A = zeros(49,49);
A(1,1) = (h^2)*(x(1)^2)-2;
A(1,2) = 1;
A(49,48) = 1;
A(49,49) = (h^2)*(x(49)^2)-1;
for m = 2:48
    for n = 1:49
        A(m,n) = 0;
        if (n == (m-1))
            A(m,n) = 1;
        elseif (n == m)
            A(m,n) = (h^2)*(x(m)^2)-2;
        elseif (n == m+1)
            A(m,n) = 1;
        end
    end
end
b = zeros(49);
b(1) = -1;
b(49) = -0.5*h;
u1 = inv(A)*b';
u(1) = 1;
u(51) = u1(49)+0.5*h;
for k = 2:50
    u(k) = u1(k-1);
end
plot(xplot,u);xlabel('x');ylabel('u(x)')
```

Prob 2

Part (a): The derivation of the finite difference formula was given in class. Solution:



Matlab code in next page

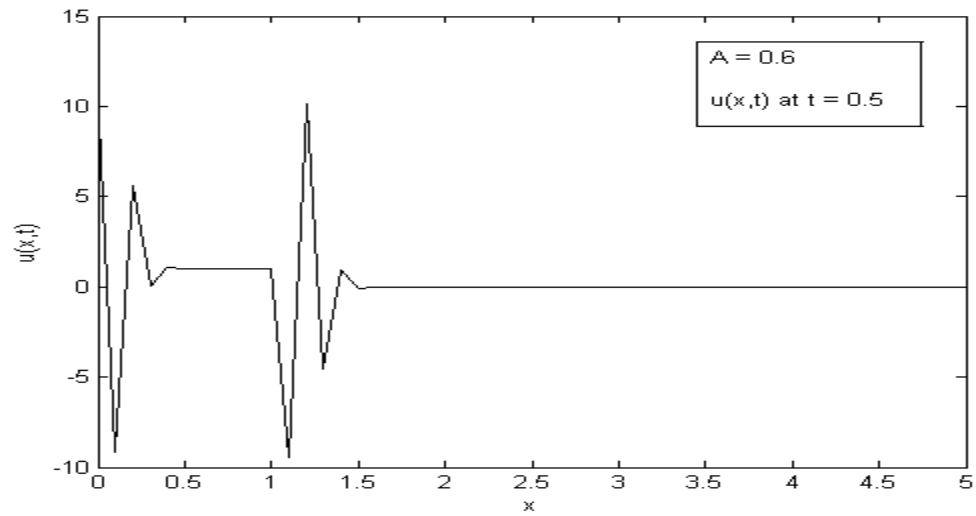
Matlab code for Prob 2(a)

```
dx = 0.1; dt = 0.1; A = -0.6;
x = [-0.1:0.1:5.1];
xx = [0:0.1:5];
for k = 1:52
    if (x(k) >= 0) && (x(k) <= 1)
        u(k) = 1;
    else
        u(k) = 0;
    end
end
for k = 2:52
    u00(k-1) = u(k);
end
for n = 1:5
    for k = 2:52
        u1(k) = (1+A*dt/dx)*u(k) - (A*dt/dx)*u(k-1);
    end
    u1(1) = 0;
    for k = 1:52
        u(k) = u1(k);
    end
end
for k = 2:52
    u05(k-1) = u(k);
end
for n = 1:5
    for k = 2:52
        u1(k) = (1+A*dt/dx)*u(k) - (A*dt/dx)*u(k-1);
    end
    u1(1) = 0;
    for k = 1:52
        u(k) = u1(k);
    end
end
for k = 2:52
    u10(k-1) = u(k);
end
for n = 1:10
    for k = 2:52
        u1(k) = (1+A*dt/dx)*u(k) - (A*dt/dx)*u(k-1);
    end
    u1(1) = 0;
    for k = 1:52
        u(k) = u1(k);
    end
end
for k = 2:52
    u20(k-1) = u(k);
end
plot(xx,u00,'k-',xx,u05,'r-',xx,u10,'b-',xx,u20,'m-')
axis([0 5 0 1.2])
legend('t = 0','t = 0.5','t = 1','t = 2')
xlabel('x');ylabel('u(x,t)')
```

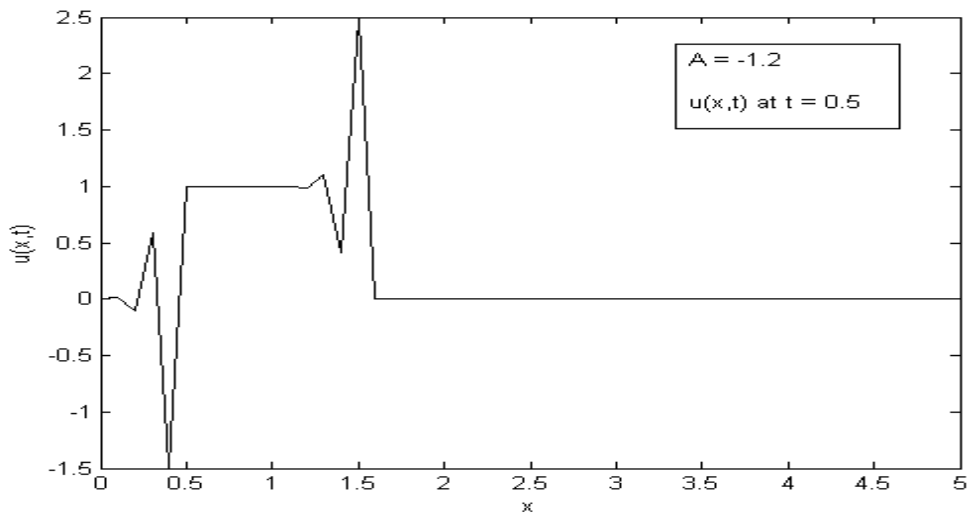
Prob 2 Part (b)

Results after integrating the system for a few steps:

$A = 0.6$



$A = -1.2$



The reasons that the numerical scheme does not work for these cases were discussed in class.

Prob 3

We have discussed the detail of the solutions in class.