MAE384 Fall 2012 HW1 Solutions

Prob 1 (prepared by HPH)

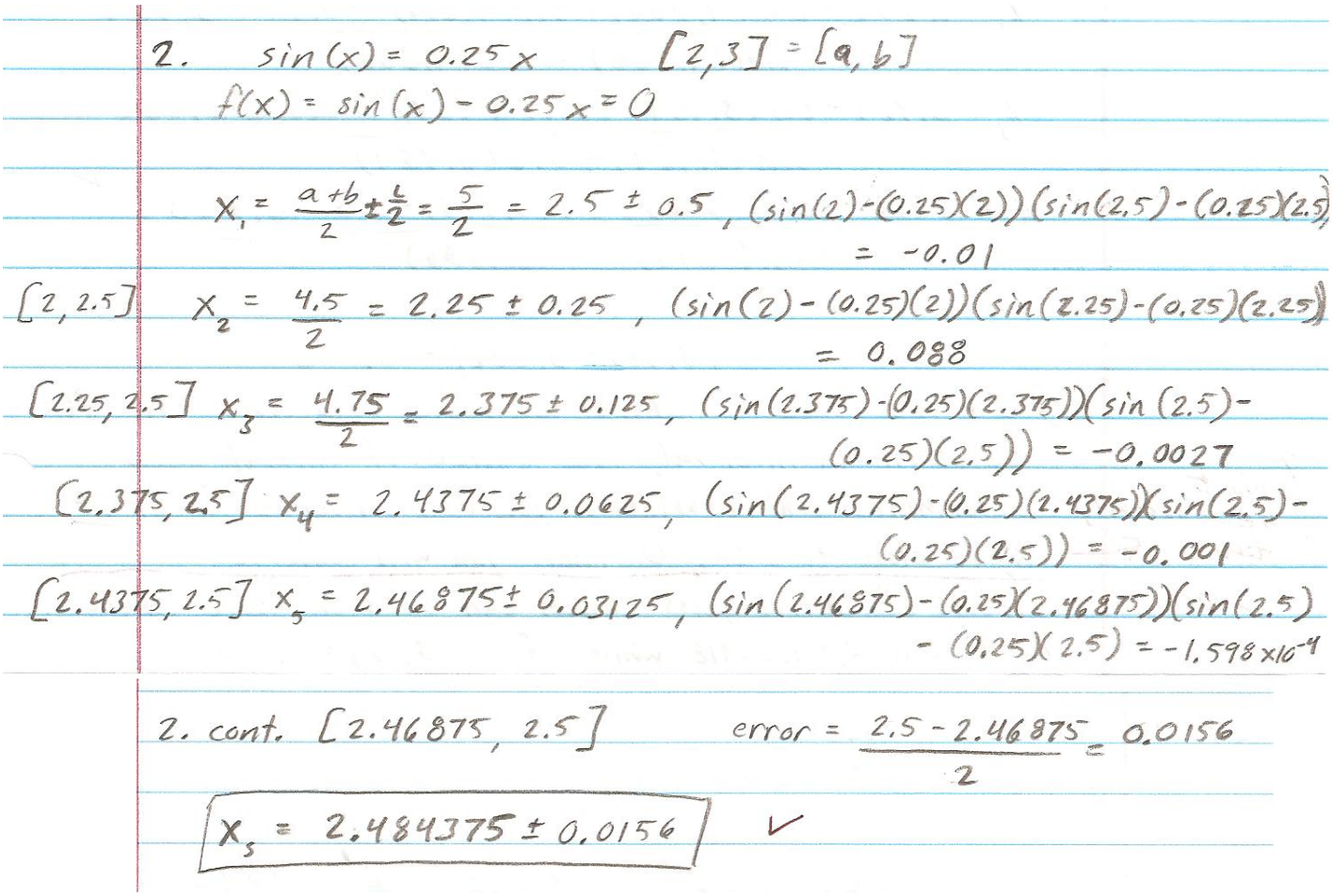
(a) The answer is $1.84375 \times 2^{19} = 966656$

(b) The smallest positive number allowed in Matlab is 2^(-1074), or 4.940656458412465e-324. Matlab will underflow (return "0") if an operation leads to a number that is less than or equal to 2^(-1075). Note that 2^(-1074) is the "resolution" of the 64-bit system in Matlab. There is literally nothing between 2(-1074) and 2^(-1075). Thus, if you enter 2^(-1074.99) (as some of you did) Matlab would still give you the same number as 2^(-1074), or 4.940656458412465e-324.

Prob 2 (prepared by HPH)

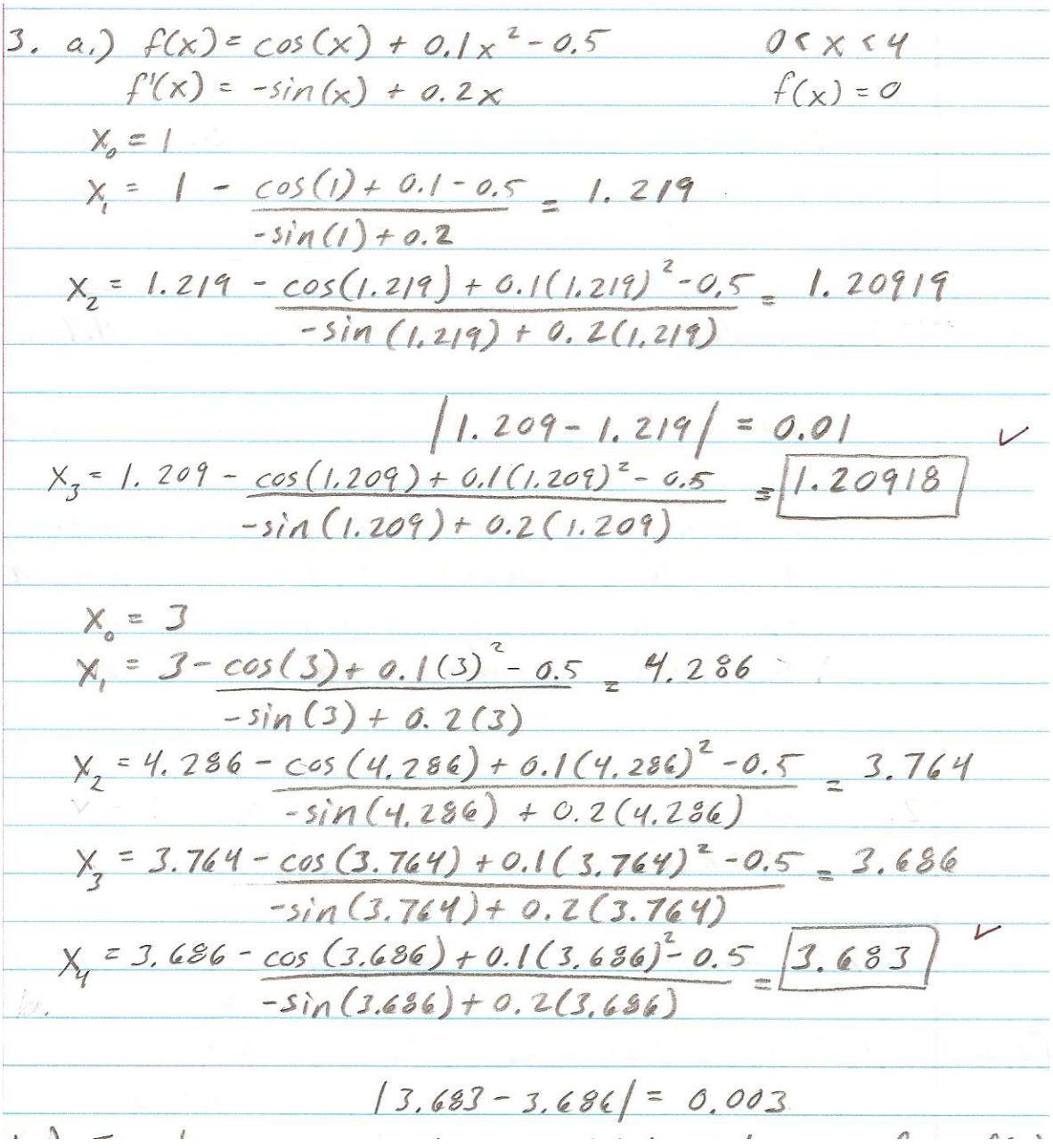
The numerical solution is $X_{NS} = 2.484375 \pm 0.015625$. We obtain it after the interval has been narrowed down to [2.46875, 2.5]. If we pick the mid-point of that interval as the solution, $X_{NS} = (2.46875+2.5)/2 = 2.484375$, the numerical error is guaranteed to be within $\pm |(2.5 - 2.46875)/2| = \pm 0.015625$. (If this is not clear to you, it's useful to read the discussion related to Eq. (3.6) and (3.7) in textbook.)

Prob 2 An example of solution by hand (Thanks to Nolan Cheshire)



Prob 3a (Prepared by HPH)
The solutions within (0, 4) are X = 1.20918 and 3.68343. See Part (b) for detail.

Prob 3a An example of solution by hand (Thanks to Nolan Cheshire)



3. a.) $f(x) = \cos(x) + 0.1x^2 - 0.5$ $\quad 0 < x < 4$

$f'(x) = -\sin(x) + 0.2x$ $\quad f(x) = 0$

$X_0 = 1$

$X_1 = 1 - \dfrac{\cos(1) + 0.1 - 0.5}{-\sin(1) + 0.2} = 1.219$

$X_2 = 1.219 - \dfrac{\cos(1.219) + 0.1(1.219)^2 - 0.5}{-\sin(1.219) + 0.2(1.219)} = 1.20919$

$|1.209 - 1.219| = 0.01$ ✓

$X_3 = 1.209 - \dfrac{\cos(1.209) + 0.1(1.209)^2 - 0.5}{-\sin(1.209) + 0.2(1.209)} = \boxed{1.20918}$

$X_0 = 3$

$X_1 = 3 - \dfrac{\cos(3) + 0.1(3)^2 - 0.5}{-\sin(3) + 0.2(3)} = 4.286$

$X_2 = 4.286 - \dfrac{\cos(4.286) + 0.1(4.286)^2 - 0.5}{-\sin(4.286) + 0.2(4.286)} = 3.764$

$X_3 = 3.764 - \dfrac{\cos(3.764) + 0.1(3.764)^2 - 0.5}{-\sin(3.764) + 0.2(3.764)} = 3.686$

$X_4 = 3.686 - \dfrac{\cos(3.686) + 0.1(3.686)^2 - 0.5}{-\sin(3.686) + 0.2(3.686)} = \boxed{3.683}$ ✓

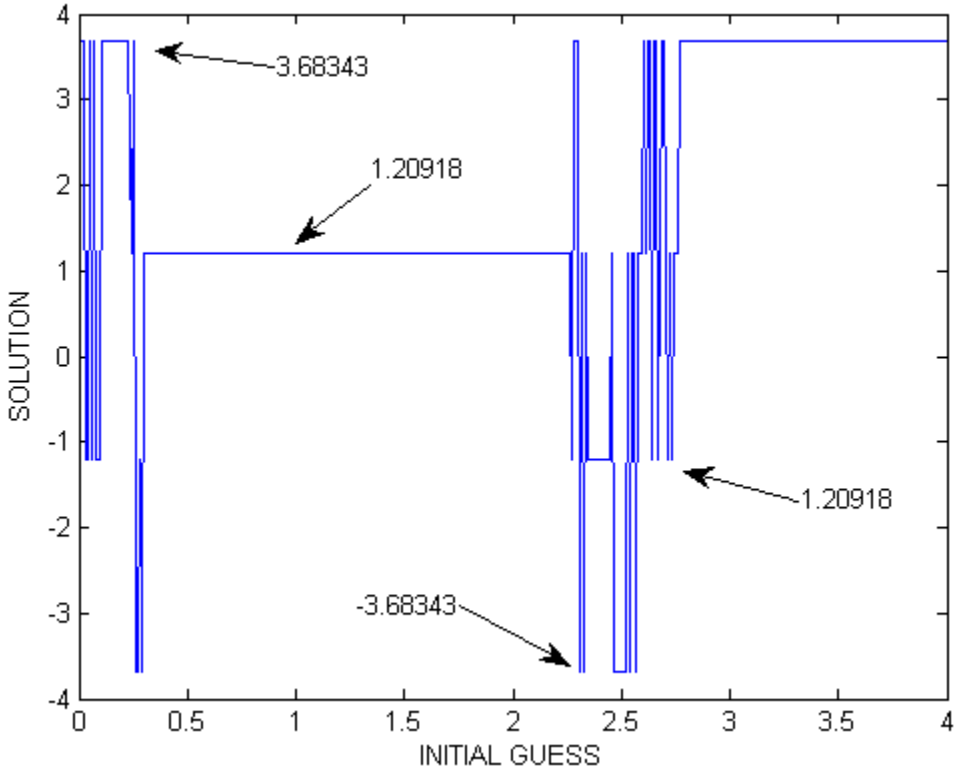
$|3.683 - 3.686| = 0.003$

Prob 3b (Prepared by HPH. This discussion also synthesizes the insights from the solutions of many students. Thanks especially to Joel Richardson, Daniel Miskin, Joseph Williams, Roaldi Joco, Jonathan Lai, and Ali Alnazawi. Apology to anyone who is left out.)

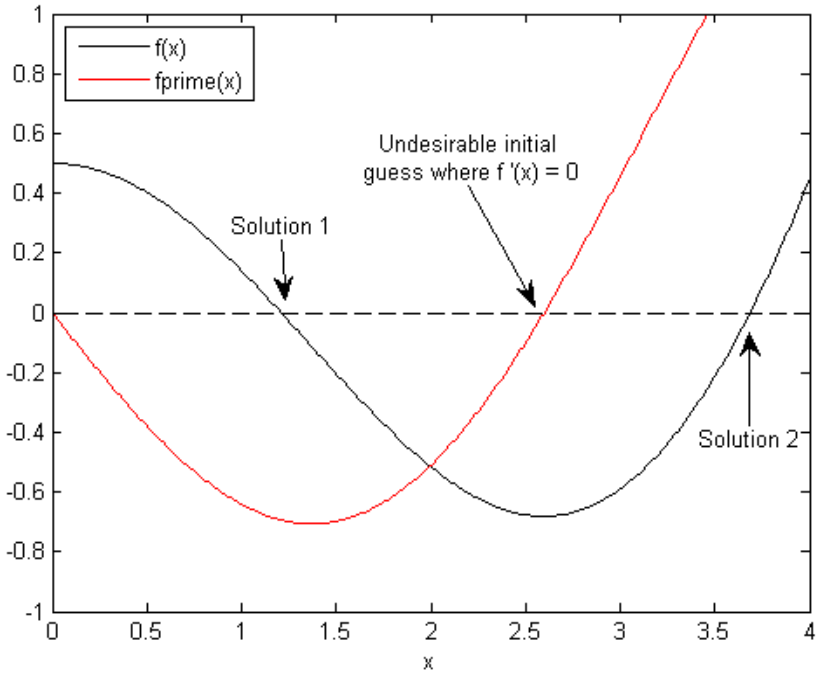
We use the following Matlab program to "scan" the solution when the initial guess is systematically varied from 0.01 to 4, step 0.01:

```
f = inline('cos(x)+0.1*x^2-0.5','x');
fprime = inline('-sin(x)+0.2*x','x');
for k = 1:400
    x0 = k*0.01;
    x00 = x0;
    for iter = 1:100
        x1 = x0 - f(x0)/fprime(x0);
        x0 = x1;
    end
    fprintf('initial guess = %8.5f   solution = %8.5f \r',x00,x1)
    xplot(k) = x00; splot(k) = x1;
end
plot(xplot,splot)
xlabel('INITIAL GUESS'); ylabel('SOLUTION')
```

The result is presented as a plot of the initial guess vs. final solution:



There are 4 solutions for the equation: 1.20918, 3.68343, -1.20918, and -3.68343 (the first two are the positive solutions for Part (a)). The majority of the initial guesses from [0, 4] converge to either 1.20918 or 3.68343. However, when the initial guess is close to where f '(x) = 0, the behavior of convergence becomes very complicated. Some initial guesses converge to the negative solutions. For a quick check, the plot in the next page shows where f '(x) = 0 (the black and red curves are f(x) and f '(x)).



The matlab program in the preceding page also gives us a print out of the solutions, as detailed below. The list is truncated to focus on the most interesting sub-intervals, and the two negative solutions are colored in red and blue:

initial guess = 0.01000 solution = 3.68343
initial guess = 0.02000 solution = 3.68343
initial guess = 0.03000 solution = -1.20918
initial guess = 0.04000 solution = -1.20918
initial guess = 0.05000 solution = 3.68343
initial guess = 0.06000 solution = -1.20918
initial guess = 0.07000 solution = 3.68343
initial guess = 0.08000 solution = -1.20918
initial guess = 0.09000 solution = -1.20918
initial guess = 0.10000 solution = -1.20918
initial guess = 0.11000 solution = 3.68343
initial guess = 0.12000 solution = 3.68343
. . . ← All 3.68343 within this range
initial guess = 0.22000 solution = 3.68343
initial guess = 0.23000 solution = 3.68343
initial guess = 0.24000 solution = 1.20918
initial guess = 0.25000 solution = 3.68343
initial guess = 0.26000 solution = -3.68343
initial guess = 0.27000 solution = -3.68343
initial guess = 0.28000 solution = -1.20918
initial guess = 0.29000 solution = -3.68343
initial guess = 0.30000 solution = 1.20918
initial guess = 0.31000 solution = 1.20918
. . . ← All 1.20918 within this range
initial guess = 2.25000 solution = 1.20918
initial guess = 2.26000 solution = 1.20918
initial guess = 2.27000 solution = -1.20918
initial guess = 2.28000 solution = 3.68343
initial guess = 2.29000 solution = 3.68343
initial guess = 2.30000 solution = 3.68343

initial guess = 2.31000 solution = -3.68343
initial guess = 2.32000 solution = 1.20918
initial guess = 2.33000 solution = -3.68343
initial guess = 2.34000 solution = 1.20918
initial guess = 2.35000 solution = -1.20918
initial guess = 2.36000 solution = -1.20918
initial guess = 2.37000 solution = -1.20918
initial guess = 2.38000 solution = -1.20918
initial guess = 2.39000 solution = -1.20918
initial guess = 2.40000 solution = -1.20918
initial guess = 2.41000 solution = -1.20918
initial guess = 2.42000 solution = -1.20918
initial guess = 2.43000 solution = -1.20918
initial guess = 2.44000 solution = -1.20918
initial guess = 2.45000 solution = -1.20918
initial guess = 2.46000 solution = 1.20918
initial guess = 2.47000 solution = -3.68343
initial guess = 2.48000 solution = -3.68343
initial guess = 2.49000 solution = -3.68343
initial guess = 2.50000 solution = -3.68343
initial guess = 2.51000 solution = -3.68343
initial guess = 2.52000 solution = -3.68343
initial guess = 2.53000 solution = 1.20918
initial guess = 2.54000 solution = -3.68343
initial guess = 2.55000 solution = 1.20918
initial guess = 2.56000 solution = 1.20918
initial guess = 2.57000 solution = -3.68343
initial guess = 2.58000 solution = 1.20918
initial guess = 2.59000 solution = 1.20918
initial guess = 2.60000 solution = 3.68343
initial guess = 2.61000 solution = 1.20918
initial guess = 2.62000 solution = 3.68343
initial guess = 2.63000 solution = 3.68343
initial guess = 2.64000 solution = -1.20918
initial guess = 2.65000 solution = 3.68343
initial guess = 2.66000 solution = 3.68343
initial guess = 2.67000 solution = -1.20918
initial guess = 2.68000 solution = 1.20918
initial guess = 2.69000 solution = 3.68343
initial guess = 2.70000 solution = 3.68343
initial guess = 2.71000 solution = -1.20918
initial guess = 2.72000 solution = 1.20918
initial guess = 2.73000 solution = -1.20918
initial guess = 2.74000 solution = 1.20918
initial guess = 2.75000 solution = 1.20918
initial guess = 2.76000 solution = 1.20918
initial guess = 2.77000 solution = 3.68343
initial guess = 2.78000 solution = 3.68343
. . . ← All 3.68343 within this range
initial guess = 3.99000 solution = 3.68343
initial guess = 4.00000 solution = 3.68343