MAE384 Fall 2012 Homework #2

1. Using the *Fixed-point iteration method*, find all solutions for the equation,

$$1 - x^3 + 2\sin(x) - \frac{1}{x} = 0 \quad .$$

For each solution you obtain, please clearly describe your choice of the "g(x)" used in the iterative formula "x = g(x)", and the corresponding initial guess to start the iteration. No credit will be given without this information even if your solution appears to be correct. (Hint: There are total of two solutions.) [**3 points**]

2. A system of linear equations is given as

$$\begin{aligned} 4x_1 + \ \ 2x_2 + \ \ \ \ x_3 &= 10 \\ 3x_1 + \ \ 5x_2 + 0.5x_3 &= 14 \\ x_1 + 0.5x_2 + \ \ 3x_3 &= 8 \end{aligned} \quad . \qquad \text{Eq. (1)}$$

(a) Solve the system by *Gauss-Jordan elimination*. Show your procedure. Save the outcome of this part as the "exact" solution, $\boldsymbol{x}_S$.
(b) Solve the system by the *Jacobi iterative method*, using (x_1, x_2, x_3) = (0,0,0) as the initial guess and perform 5 iterations. Here, one iteration means updating all x_1, x_2, and x_3 once.
(c) Solve the system by the *Gauss-Seidel iterative method*, using (x_2, x_3) = (0,0) as the initial guess. Perform 5 iterations. Again, one iteration means updating all x_1, x_2, and x_3 once (see Example 4-8 in textbook).
(d) From the outcome of (b) and (c), calculate and plot the numerical error as a function of the number of iteration for both Jacobi and Gauss-Seidel methods. The error is defined using the *Euclidean 2-norm* (see Eq. 4.72) as

$$E \equiv \|\boldsymbol{x_N} - \boldsymbol{x_S}\| \equiv \sqrt{(x_{N,1}-x_{S,1})^2 + (x_{N,2}-x_{S,2})^2 + (x_{N,3}-x_{S,3})^2} \quad ,$$

where $\boldsymbol{x}_N \equiv (x_{N,1}, x_{N,2}, x_{N,3})$ is the numerical solution from the iterative method and $\boldsymbol{x}_S \equiv (x_{S,1}, x_{S,2}, x_{S,3})$ is the "exact" solution obtained by Gauss-Jordan elimination from Part (a). Comment on your result.

(e) Equation (1) can be written in matrix form as

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \, ,$$

where

$$\boldsymbol{A} \equiv \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & 0.5 \\ 1 & 0.5 & 3 \end{pmatrix} , \quad \boldsymbol{x} \equiv \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} , \text{ and } \quad \boldsymbol{b} \equiv \begin{pmatrix} 10 \\ 14 \\ 8 \end{pmatrix} .$$

Using the *Euclidean norm* for matrix (Eq. 4.76 in textbook), evaluate the *condition number* for the system in Eq. (1). Is this system ill-conditioned? **[5 points]**