MAE384 Fall 2012 Homework #4

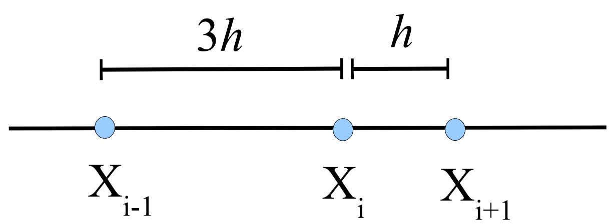
In all problems, the argument of a sinusoidal function is always in <u>radian</u>.

1. Evaluate the first derivative of the function, $f(x) = \cos(x^3)$, for the interval $0 \le x \le 3$. First, find $f'(x)$ analytically to prepare for later discussions.
(a) Evaluate $f'(x)$ at the discrete points of $x = 0, 0.1, 0.2, ..., 2.9, 3.0$ by setting $h = 0.1$ and using the following two formulas: (i) The 2-point forward difference scheme (1st formula in Table 6-1 in p. 259), and (ii) The 3-point forward difference scheme (2nd formula in Table 6-1 in p. 259). Plot the numerical results and analytic solution (total of 3 curves) in a single figure.
(b) Repeat (a) but now set $h = 0.01$ and evaluate $f'(x)$ at $x = 0, 0.01, 0.02, ..., 2.99, 3.0$.
(c) Discuss the results in (a) and (b). **[4 points]**

2. Consider the non-uniform grid (shown in the diagram below) with $x_i - x_{i-1} = 3h$ and $x_{i+1} - x_i = h$. Derive a 3-point finite difference formula for the <u>first derivative</u> of $f(x)$ at $x = x_i$ that has a truncation error of $O(h^2)$. Your formula should have the form:

$$f'(x_i) = A\,f(x_{i-1}) + B\,f(x_i) + C\,f(x_{i+1}) + O(h^2) \ .$$

Please clearly describe what your A, B, and C are in the final answer. **[2 points]**



3. All of the formula in Table 6-1 have a truncation error of $O(h)$, $O(h^2)$, or $O(h^4)$. Try to derive a five-point finite difference formula for the <u>second derivative</u> of $f(x)$ at $x = x_i$ that has a truncation error of $O(h^3)$. Moreover, the formula must have the following form:

$$f''(x_i) = A\,f(x_{i-1}) + B\,f(x_i) + C\,f(x_{i+1}) + D\,f(x_{i+2}) + E\,f(x_{i+3}) + O(h^3) \ .$$

In other words, the five points must include x_i itself, one point to its left, and three points to its right. The spacing between two adjacent grid points is h = constant. After solving the problem, clearly state what your A, B, C, D, and E are. **[4 points]**