

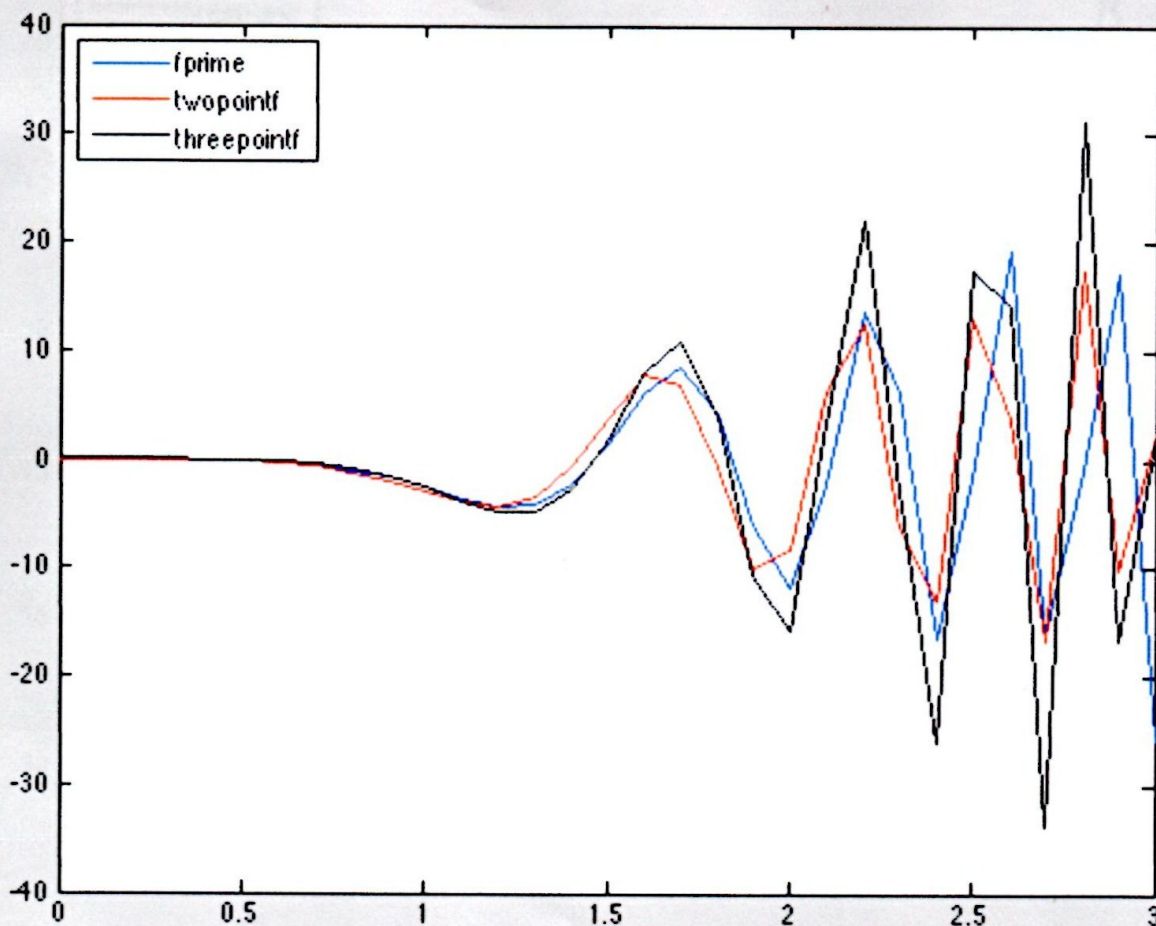
**Prob 1** (Thanks to Ali Alnazawi)

Part (a)

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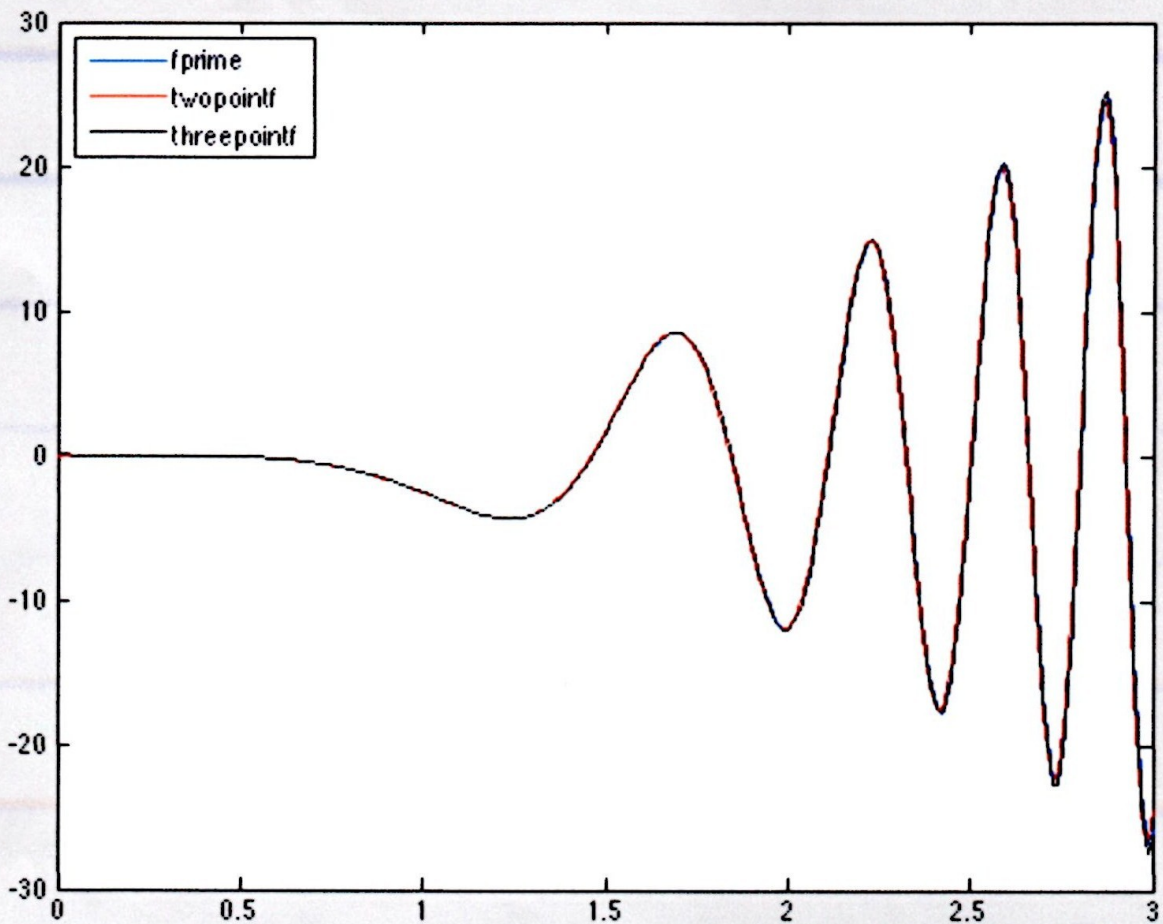
% d/dx cos(x^3) = -3*(x^2)*sin(x^3)
% Part (a)
x=0:0.1:3;
fprime = -3*(x.^2).*sin(x.^3);
plot(x,fprime,'b-')
hold on
x2= 0:0.1:3;
h=0.1;
twopointf= (cos((x2+0.1).^3)-cos(x2.^3))/h;
plot(x2,twopointf,'r-')
x3=0:0.1:3;
threepointf= (-3*(cos(x3.^3))+4*(cos((x3+0.1).^3))-
cos((x3+0.2).^3))/(2*h);
plot(x3,threepointf,'k-')
legend('fprime','twopointf','threepointf','location','northwest')

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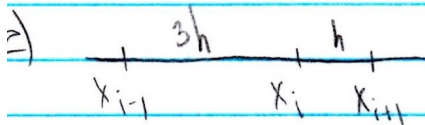


Prob1 Part (b)

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% part (b)
x=0:0.01:3;
fprime = -3*(x.^2).*sin(x.^3);
plot(x,fprime,'b-')
hold on
x2= 0:0.01:3;
h=0.01;
twopointf= (cos((x2+0.01).^3)-cos(x2.^3))/h;
plot(x2,twopointf,'r-')
x3=0:0.01:3;
threepointf= (-3*(cos(x3.^3))+4*(cos((x3+0.01).^3))-
cos((x3+0.02).^3))/(2*h);
plot(x3,threepointf,'k-')
legend('fprime','twopointf','threepointf','location','northwest')
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Prob 2 (Thanks to Bauyrzhan Alibekov)



$$f(x_{i+1}) = f(x_i) + f'(x_i)h + f''(x_i)\frac{h^2}{2} + f'''(x_i)\frac{h^3}{6} + \dots$$

$$f(x_{i-1}) = f(x_i) - 3f'(x_i)h + f''(x_i)\left(\frac{9h^2}{2}\right) - f'''(x_i)\left(\frac{27h^3}{6}\right) + \dots \quad \Big| \times \left(-\frac{1}{9}\right)$$

$$f(x_{i+1}) - \frac{1}{9}f(x_{i-1}) = \frac{8}{9}f(x_i) + \frac{4}{3}f'(x_i)h + 0 + O(h^3)$$

$$f'(x_i) = \frac{-\frac{1}{9}f(x_{i-1}) - \frac{8}{9}f(x_i) + f(x_{i+1}))}{\frac{4h}{3}} + O(h^2) =$$

$$= \boxed{-\frac{1}{12h}f(x_{i-1}) - \frac{2}{3h}f(x_i) + \frac{3}{4h}f(x_{i+1}) + O(h^2)}$$

$$A = -\frac{1}{12h} \quad B = -\frac{2}{3h} \quad C = \frac{3}{4h}$$

✓

Prob 3 (Thanks to Bauyrzhan Alibekov)

$$I \quad f(x_{i+1}) = f(x_i) + f'(x_i)h + f''(x_i)\frac{h^2}{2} + f'''(x_i)\frac{h^3}{6} + f^{(4)}(x_i)\frac{h^4}{24} + f^{(5)}(x_i)\frac{h^5}{120}$$

$$J \quad f(x_{i+2}) = f(x_i) + f'(x_i)2h + f''(x_i)2h^2 + f'''(x_i)\frac{4}{3}h^3 + f^{(4)}(x_i)\frac{2}{3}h^4 + f^{(5)}(x_i)\frac{4}{15}h^5$$

$$K \quad f(x_{i+3}) = f(x_i) + f'(x_i)3h + f''(x_i)\frac{9}{2}h^2 + f'''(x_i)\frac{9}{2}h^3 + f^{(4)}(x_i)\frac{27}{8}h^4 + f^{(5)}(x_i)\frac{81}{40}h^5$$

$$L \quad f(x_{i-1}) = f(x_i) - f'(x_i)h + f''(x_i)\frac{h^2}{2} - f'''(x_i)\frac{h^3}{6} + f^{(4)}(x_i)\frac{h^4}{24} - f^{(5)}(x_i)\frac{h^5}{120}$$

$$I + 2J + 3K - L = 0$$

$$\frac{1}{6} + \frac{4}{3}J + \frac{9}{2}K - \frac{1}{6}L = 0$$

$$\frac{1}{24} + \frac{2}{3}J + \frac{27}{8}K + \frac{1}{24}L = 0$$

$$\Rightarrow \begin{pmatrix} 2 & 3 & -1 \\ 4/3 & 9/2 & -1/6 \\ 2/3 & 27/8 & 1/24 \end{pmatrix} \begin{pmatrix} J \\ K \\ L \end{pmatrix} = \begin{pmatrix} -1 \\ -1/6 \\ -1/24 \end{pmatrix}$$

$$J = \frac{2}{3} \quad K = -\frac{1}{6} \quad L = \frac{1}{6}$$

$$f(x_{i+1}) + \frac{2}{3}f(x_{i+2}) - \frac{1}{6}f(x_{i+3}) + \frac{11}{6}f(x_{i-1}) = \frac{10}{3}f(x_i) + f''(x_i)\left(\frac{h^2}{2} + \frac{4h^2}{3} - \frac{3h^2}{4} + \frac{11h^2}{12}\right) + O(h^5)$$

$$f''(x_i) = \frac{\frac{11}{6}f(x_{i-1}) - \frac{10}{3}f(x_i) + f(x_{i+1}) + \frac{2}{3}f(x_{i+2}) - \frac{1}{6}f(x_{i+3})}{2h^2} + O(h^3) =$$

$$= \left[ \frac{11}{12h^2}f(x_{i-1}) - \frac{5}{3h^2}f(x_i) + \frac{1}{2h^2}f(x_{i+1}) + \frac{1}{3h^2}f(x_{i+2}) - \frac{1}{12h^2}f(x_{i+3}) + O(h^3) \right]$$

$$A = \frac{11}{12h^2}$$

$$B = -\frac{5}{3h^2}$$

$$C = \frac{1}{2h^2}$$

$$D = \frac{1}{3h^2}$$

$$E = -\frac{1}{12h^2}$$