MAE384 Fall 2012 HW5

In all problems, the argument of a sinusoidal function is in radian.

Unless you solve the problem(s) by hand, please submit the computer codes that produce your numerical solutions.

1. Evaluate the following integral,

$$I = \int_0^6 e^x \sin(e^x)\,dx \quad ,$$

using **(a)** the Trapezoidal method with h = 0.01 and 0.001, and **(b)** the composite Simpson's 3/8 method with h = 0.01, and 0.001. Compare the four values obtained by numerical integration with the analytic solution. **[2 points]**

3. (a) Solve the following initial value problem,

$$\frac{du}{dx} = x^2\, e^{-(u+1)} \quad , u(0) = 1,$$

using the classical 4rd order Runge-Kutta method with h = 0.3. Find the solution $u(x)$ for the interval of $0 \leq x \leq 3$. Also, solve the initial value problem analytically, then compare the numerical and analytic solutions by plotting them together. **[3 points]**

(b) Same as **(a)** but use Euler's explicit method with h = 0.3 to find the numerical solution for $0 \leq x \leq 3$. Compare the numerical and analytic solutions by plotting them in a single figure. **[2 points]**

4. Solve the initial value problem,

$$\frac{d^2u}{dx^2} + 5\frac{du}{dx} + 6u = 0 \quad ,$$

with the initial condition: (I) $u(0) = 2$, (II) $u'(0) = -5$ (u' is du/dx) ,

using the following method: (i) The 3-point central difference scheme (9th formula from top in p. 260) is used to represent u'' in the ODE. (ii) The 2-point forward difference scheme (1st formula in Table 6-1 in p. 259) is used to represent u' in the ODE and in the 2nd initial condition. Find the numerical solution for the interval, $0 \leq x \leq 1.5$, for the two cases: h = 0.1 and h = 0.05. (In each case, use the same h in (i) and (ii)). Find the analytic solution and compare it with the numerical solutions. **[2 points]**