MAE384 Fall 2012 HW5 solutions

Prob 1 (prepared by HPH)

```
clear
f = inline('exp(x)*sin(exp(x))','x');
I_exact = cos(1)-cos(exp(6));
% ---- Trapezoidal with h = 0.01 ----
h = 0.01; N = 6/h;
I_trap_01 = (f(0)+f(6))/2;
for j = 1:(N-1)
    I_trap_01 = I_trap_01 + f(j*h);
end
I_trap_01 = I_trap_01*h;
% ---- Trapezoidal with h = 0.001 ----
h = 0.001; N = 6/h;
I_trap_001 = (f(0)+f(6))/2;
for j = 1:(N-1)
    I_trap_001 = I_trap_001 + f(j*h);
end
I_trap_001 = I_trap_001*h;
% ---- Simpson 3/8 with h = 0.01 ----
h = 0.01; N = 6/h;
I_sim38_01 = f(0)+f(6);
for j = 1:(N-1)
    if (mod(j,3) == 0)
        I_sim38_01 = I_sim38_01 + 2*f(j*h);
    else
        I_sim38_01 = I_sim38_01 + 3*f(j*h);
    end
end
I_sim38_01 = I_sim38_01*(3*h/8);
% ---- Simpson 3/8 with h = 0.001 ----
h = 0.001; N = 6/h;
I_sim38_001 = f(0)+f(6);
for j = 1:(N-1)
    if (mod(j,3) == 0)
        I_sim38_001 = I_sim38_001 + 2*f(j*h);
    else
        I_sim38_001 = I_sim38_001 + 3*f(j*h);
    end
end
I_sim38_001 = I_sim38_001*(3*h/8);
%
I_exact
I_trap_01
I_trap_001
I_sim38_01
I_sim38_001
```

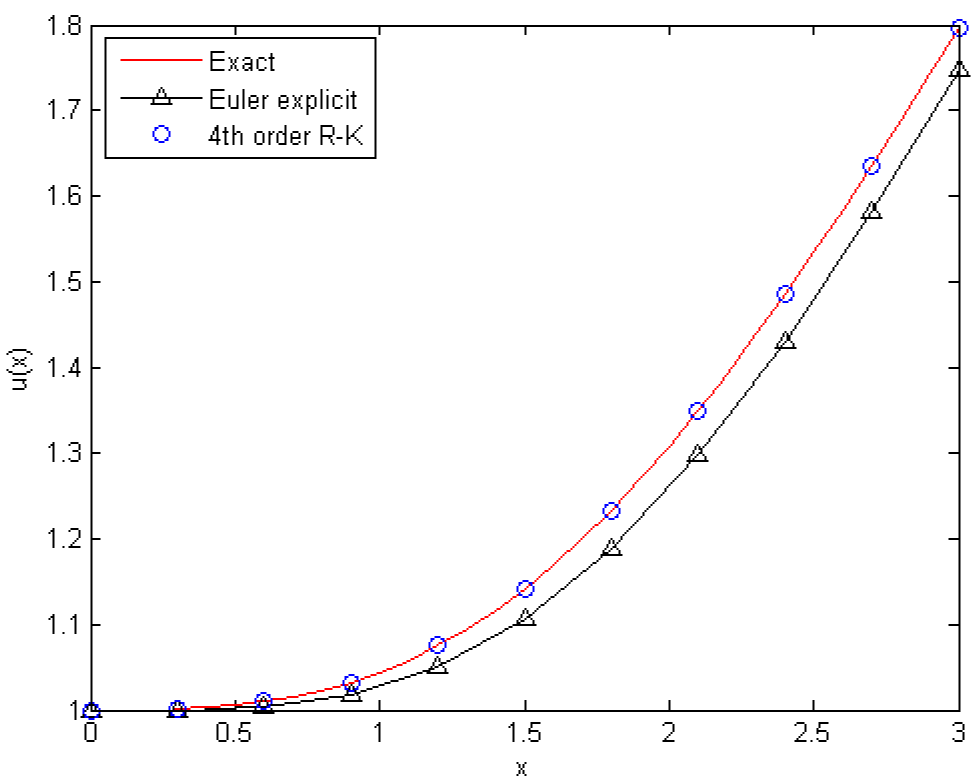
Answer:

I_exact	=	0.2776	Exact
I_trap_01	=	0.8172	Trapezoidal, h = 0.01
I_trap_001	=	0.2812	Trapezoidal, h = 0.001
I_sim38_01	=	-4.7022	Simpson's 3/8, h = 0.01
I_sim38_001	=	0.2775	Simpson's 3/8, h = 0.001

Prob 2 (prepared by HPH)

Analytic solution is $u(x) = \ln(\frac{x^3}{3} + e^2) - 1$.

```
clear
f = inline('(x^2)*exp(-(u+1))','x','u');
% ---- Euler explicit ----
x1 = [0:0.3:3];
h = 0.3;
N = 3/h;
u1(1) = 1;
for i = 1:N
    u1(i+1) = u1(i)+h*f(x1(i),u1(i));
end
% ---- 4th order R-K ----
x2 = [0:0.3:3];
h = 0.3;
N = 3/h;
u2(1) = 1;
for i = 1:N
    K1 = f(x2(i),u2(i));
    K2 = f(x2(i)+h/2, u2(i)+K1*h/2);
    K3 = f(x2(i)+h/2, u2(i)+K2*h/2);
    K4 = f(x2(i)+h, u2(i)+K3*h);
    u2(i+1) = u2(i)+h*(K1+2*K2+2*K3+K4)/6;
end
% ---- Exact ----
x = [0:0.1:3];
u_exact = log((x.^3)/3 + exp(2)) - 1;
% ---- making plot ----
plot(x,u_exact,'r-',x1,u1,'k-^',x2,u2,'bo')
legend('Exact','Euler explicit','4th order R-K','Location','NorthWest')
xlabel('x');ylabel('u(x)')
```



Prob 3 (prepared by HPH)

Analytic solution is $e^{-2x}+e^{-3x}$.

Finite difference formula for the ODE: $\dfrac{u_{i+1}-2u_i+u_{i-1}}{h^2}+5\,\dfrac{u_{i+1}-u_i}{h}+6\,u_i=0$

$$\Longrightarrow \quad u_{i+1}=\frac{2+5h-6h^2}{1+5h}\,u_i-\frac{1}{1+5h}\,u_{i-1}$$

Initial conditions: $u_0=2$ and $\dfrac{u_1-u_0}{h}=-5 \Longrightarrow u_1=u_0-5h$.

```
clear
% ---- Numerical solution, h = 0.1 ----
h = 0.1; N = 1.5/h + 1;
xplot1 = [0:h:1.5];
u1(1) = 2; u1(2) = u1(1)-5*h;
c1 = (2+5*h-6*(h^2))/(1+5*h); c2 = -1/(1+5*h);
for k = 3:N
    u1(k) = c1*u1(k-1)+c2*u1(k-2);
end
% ---- Numerical solution, h = 0.05 ----
h = 0.05; N = 1.5/h + 1;
xplot2 = [0:h:1.5];
u2(1) = 2; u2(2) = u2(1)-5*h;
c1 = (2+5*h-6*(h^2))/(1+5*h); c2 = -1/(1+5*h);
for k = 3:N
    u2(k) = c1*u2(k-1)+c2*u2(k-2);
end
% ---- Exact solution ----
x = [0:0.05:1.5];
u_exact = exp(-3.*x) + exp(-2.*x);
plot(x,u_exact,'k-',xplot1,u1,'r-',xplot2,u2,'b-')
xlabel('x');ylabel('u(x)')
legend('Exact','h = 0.1','h = 0.05')
```