MAE384 Fall 2012 Homework #6

1. Solve the boundary value problem,

$$\frac{d^2 u}{d x^2} + (15+8x)\, u = 0 \quad ,$$

$$u'(0) = 0.5 \ , \ \ u(1) = 0.3 \ ,$$

for $u(x)$ over the interval of $0 \le x \le 1$. Use the 3-point central difference scheme (9th formula from top in p. 260) to represent u'' in the differential equation and 2-point forward finite difference scheme (1st formula in Table 6-1 in p. 259) to represent the u' in the first boundary condition. Choose $h = 0.1$. Plot your solution. **(4 points)**

2. **(a)** Solve the partial differential equation,

$$\frac{\partial u}{\partial t} = 0.5\,\frac{\partial u}{\partial x} - 0.4\,u \quad ,$$

defined on the semi-infinite domain, $-\infty < x < \infty$ and $0 \le t < \infty$, with the boundary condition given at $t = 0$ as

$$u(x, 0) = \begin{cases} 1 & , \text{ if } 3.5 \le x \le 4 \\ 0 & , \text{ otherwise .} \end{cases}$$

Use the 2-point forward difference scheme (1st formula in Table 6-1 in p. 259) to discretize both $\partial u/\partial t$ and $\partial u/\partial x$. Choose $\Delta x = 0.1$ and $\Delta t = 0.1$. Integrate your system forward in t to find the solution, $u(x,t)$, at $t = 0.5$, 1, and 2. Plot these solutions (as a function of x) along with the "initial" state, $u(x,0)$, over the interval of $0 \le x \le 5$. **(3 points)**

3. Find the general solution of the following PDEs by the method of separation of variables. **(2 points)**

(a) $\displaystyle \frac{\partial^2 u}{\partial x\,\partial y} - x\,y\,u = 0$ **(b)** $\displaystyle x\,y\,\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + y\,u = 0$