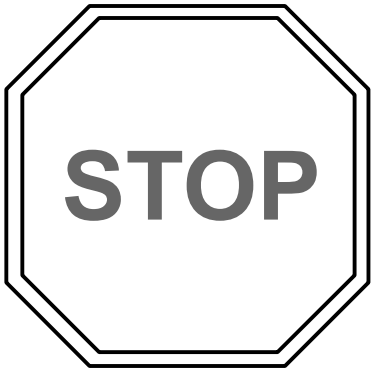


**Please read the rules before forming any collaboration for the homework. A violation of the rule(s) given in this page will be considered a violation of ASU's Academic Integrity Policy.**

Rules on collaboration for homework:

**(1) Collaboration is not allowed unless all involved follow rules (2)-(3) and unless the extent of collaboration is properly disclosed in a statement in the first page of the report for the homework. An additional instruction will be given in Homework #1 on the proper format of the statement.**

**(2) For each homework assignment, each person can have maximum of one collaborator. Be aware that a collaborator's collaborator counts as a collaborator. For example, if Alice collaborates with Bob and Bob collaborates with Charles, Charles counts as a collaborator of Alice. All three violate the rule. In other words, collaboration can only be carried out within an isolated "team of two". Please talk to a potential collaborator to ensure that this rule is not violated before establishing any collaboration.**

**(3) In a legitimate collaboration, each individual must make a non-negligible contribution to the collaborative effort. Taking the solution or computer code from another student without making a reciprocal contribution to it is not legitimate collaboration and is not allowed. To certify that a collaboration is legitimate, the contribution from each person to the collaborative effort must be documented in the mandatory statement as described in (1).**

# MAE 384, Spring 2018 Homework #1

1 point ≈ 1% of your total score for this class

Hard copy of report is due 12:15 PM on the due date. The rules on collaboration for homework will be released separately. Please follow the rules. Computer codes used to complete the tasks should be included in the report.

**Task 0** (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of report.** If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the report has not helped anyone or received help from anyone on this assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the extent of collaboration. For example:

| Name of collaborator: Joe Smith | |
|---|---|
| Task(s), specific detail | Contribution to collaborative effort |
| Task 1, Matlab code | Discussed strategy for coding and checked errors in the code with collaborator |
| Task 3, Mathematical derivation | Worked with collaborator to sort out potential choices of "g(x)" |

**Task 1 (4 points)**
Use **bisection method** to find all solutions of the equation,

$$x \sin(x^6) = 1 \ ,$$

within the interval of $1 \le x \le 2$. To receive full credit, the absolute error for each solution must be smaller than 0.0001. Please list the solutions in ascending order (i.e., from the smallest to the largest) and state clearly the numerical error for each solution.

**Task 2 (3 points)**
The following equation,

$$\exp[-(x-1)^2] + \exp[-(x-2)^2] + \exp[-(x-4)^2] - 0.9 = 0 \ ,$$

has four solutions within the interval of $0 \le x \le 5$. Suppose that **Newton's method** is used to seek the solutions, different choices of the initial guess would potentially lead to different solutions. Consider the 51 initial guesses, $x_0 = (0, 0.1, 0.2, 0.3, \ldots, 4.9, 5)$. Use Newton's method to perform 100 iterations for each of those initial guesses. List the solutions (as obtained at the end of 100 iterations) and their corresponding initial guesses in the following format:

$x_0 = 0$ solution = *.******
$x_0 = 0.1$ solution = *.******
$x_0 = 0.2$ solution = *.******
...

If for a given $x_0$ the process of iteration diverges such that Matlab returns Inf or NaN at the end of 100 iterations, please list the "Inf" or "NaN" as the solution to indicate that Newton's method fails to converge.

Briefly discuss the result. Focus on the following questions: Is there a general predictive relation between the given initial guess and the final solution? What happens when the initial guess is close to where $f'(x) = 0$? (Here, "$f(x) = 0$" describes the original equation and $f'(x)$ is the derivative of $f(x)$.) What happens when the initial guess is very close to a solution?

**Task 3 (3 points)**
**(a)** The equation,

$$x^3 + x \sin(x) - \exp(x) + 5 = 0 ,$$

has one positive solution and one negative solution. Suppose that the original equation is rearranged into the form, $x = g(x)$, and **Fixed-point iterative method** is used to seek the solutions. Moreover, a constraint is imposed that the initial guess, $x_0 = 4$, will be used to seek the positive solution, and $x_0 = -3$ will be used to seek the negative solution. Given this constraint, find an appropriate choice of $g(x)$ for each case (that targets the positive or negative solution) such that the process of iteration converges to the desired solution. Perform 100 iterations for each choice of $(x_0, g(x))$ and take the result at the end of 100 iterations as the solution. To summarize, the key task of this exercise is to fill the question marks in following table:

| Case | Given initial guess, $x_0$ (cannot be changed) | Choice of $g(x)$ that leads to convergence | Solution after 100 iterations |
|---|---|---|---|
| Target: positive solution | 4 | ? | ? |
| Target: negative solution | –3 | ? | ? |

**(b)** Is it possible to start from the initial guess, $x_0 = -3$, and use Fixed-point iterative method to find the positive solution for this problem? If "yes", provide an example of $g(x)$ that leads to convergence to the positive solution. If "no", explain why. (If the correct answer is "no", a valid explanation is required to receive full credit.)