MAE384 Spring 2018 Homework #3

Hard copy of report is due 12:15 PM on the due date. Computer codes used to complete the tasks should be included in the report.

Task 0 (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of report**. If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the report has not helped anyone or received help from anyone on this assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the extent of collaboration. Please see related clarifications in Homework #1.

Task 1. (4 points)
In this task, different numerical methods will be used to evaluate the first derivative of the function, $f(x) = x\, sin(x^6)$, over the interval of $1 \le x \le 1.8$. First, find $f'(x)$ analytically as a preparation for the following tasks.

(a) Evaluate $f'(x)$ at the discrete points of $x = 1, 1.01, 1.02, 1.03, ..., 1.79, 1.8$ by setting the grid spacing $h = 0.01$ and using the following two finite-difference formulas: (i) The 2-point central difference scheme (3rd formula in Table 8-1 in p. 318), and (ii) The 4-point central difference scheme (4th formula in Table 8-1 in p. 318). Plot the numerically evaluated and analytically derived $f'(x)$ over the interval of $1 \le x \le 1.8$. Please collect all three curves (or sets of symbols) - one analytic and two numerical - in a single plot. It is recommended that the analytic formula be plotted as connected line, and numerically obtained derivatives plotted as symbols. See Matlab Example 41-44 for useful commands for making a clear plot.

(b) Define the numerical error at a given location of x as numerically evaluated $f'(x)$ minus the exact (analytically derived) $f'(x)$. (Note: No need to take the absolute value of this difference. The error defined here can be positive or negative.) Make a plot of the numerical error as a function of x, for both numerical schemes used in Part (a). Please collect the two curves (or two sets of symbols as recommended in (a)) in the same plot.

(c) Repeat (b), but change h to 0.002 and evaluate the derivatives at $x = 1, 1.002, 1.004, 1.006, ..., 1.796, 1.798, 1.8$. Make the plot of numerical errors in the same fashion as in Part (b). (The only deliverable is the plot of the numerical errors. No need to repeat the plot of the derivatives themselves as in Part (a).)

(d) From the results of Part (b) and (c), discuss how the magnitude of numerical error changes when h is refined from 0.01 to 0.002. Does the change reflect the expectation that the two finite difference schemes are "second-order" and "fourth-order", respectively?

[Note: All plots for this task must be made over the interval of $1 \le x \le 1.8$ (including the end points) and must be properly labeled. A deduction will be assessed on plots that are of poor quality.]

Task 2. (2 points)

(a) All of the formula in Table 8-1 have a truncation error of $O(h)$, $O(h^2)$, or $O(h^4)$. Try to derive a four-point, non-centered, finite difference formula for the <u>first derivative</u> of $f(x)$ at $x = x_i$ that has a truncation error of $O(h^3)$. Moreover, the formula must have the following form:

$$f'(x_i) = \frac{A\,f(x_{i-2}) + B\,f(x_{i-1}) + C\,f(x_i) + D\,f(x_{i+1})}{h} + O(h^3) \quad .$$

In other words, the derivative of $f(x)$ at $x = x_i$ must be represented by the combination of the values of $f(x)$ at x_i itself, two neighboring grid points to its left, and one grid point to its right, as illustrated in Fig. 1. The spacing between two adjacent grid points is h = constant. <u>Show your procedure</u>. Clearly state what the values of A, B, C, and D are.

(b) After obtaining the finite difference formula in Part (a), use it to repeat Task 1(c) (i.e., setting $h = 0.002$ to evaluate the derivative of $f(x)$ in Task 1.). Plot the numerical error as a function of x in the same fashion as in Task 1(c). Given that the numerical scheme is 3rd-order, we expect it to be superior to the 2nd-order scheme but inferior to the 4th order scheme used in Task 1. Is this true based on your calculations of numerical errors? To answer this question, you may superimpose the numerical errors from Task 1(c) to the plot for the 3rd-order scheme.

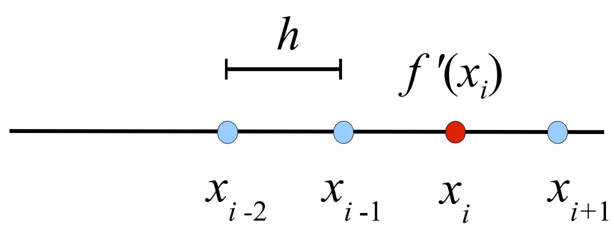

Fig. 1

Task 3 (2 points)

Derive a five-point, non-centered, finite difference formula for the <u>second derivative</u> of $f(x)$ at $x = x_i$ that has a truncation error of $O(h^3)$. Moreover, the formula must have the following form:

$$f''(x_i) = \frac{A\,f(x_i) + B\,f(x_{i+1}) + C\,f(x_{i+2}) + D\,f(x_{i+3}) + E\,f(x_{i+4})}{h^2} + O(h^3) \quad .$$

In other words, the derivative of $f(x)$ at $x = x_i$ must be represented by the combination of the values of $f(x)$ at x_i itself, and the neighboring four grid points to its right, as illustrated in Fig. 2. The spacing between two adjacent grid points is h = constant. <u>Show your procedure</u>. Clearly state the values of A, B, C, D, and E in the final answer.

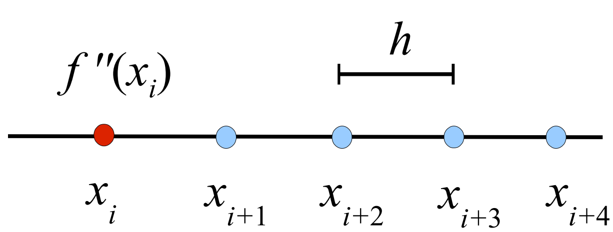

Fig. 2