MAE384, Spring 2018 Homework #5

Hard copy of report is due 12:15 PM on the due date. Computer codes used to complete the tasks should be included in the report.

Note: Uses of Matlab built-in functions for solving ODEs, such as **ode23** and **ode45**, are NOT allowed for this homework.

Task 0 (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of report**. If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the report has not helped anyone or received help from anyone on this assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the extent of collaboration. Please see related clarifications in Homework #1.

Task 1 (4 points)
Consider the following initial value problem for $u(x)$ defined on $x \geq 0$,

$$\frac{du}{dx} = -x\,u + e^{-\frac{x^2}{2}}, \quad u(0) = 1\ .$$

(a) Find the analytic solution, which will be used to validate the numerical solutions.
(b) Solve the initial value problem using the following two methods ("Δx" is "h" in the textbook):

(I) Euler's implicit method, with $\Delta x = 0.1$
(II) Third-order Runge-Kutta method, with $\Delta x = 0.1$

In both cases, numerically integrate the system to $x = 2$ to find the solution over the interval of $0 \leq x \leq 2$. Plot the analytic solution and the numerical solutions using methods (I) and (II) over the interval of $0 \leq x \leq 2$. Collect all three curves in one plot and clearly label the curves.

Task 2 (5 points)
Consider the following initial value problem for $u(x)$ defined on $x \geq 0$,

$$u'''' - u''' - 7u'' + u' + 6u = 0\ , \quad \text{("prime" is the derivative, d/d}x\text{)}$$

$$u(0) = 1.1\ ,\ u'(0) = -4.7\ ,\ u''(0) = 7.9\ ,\ u'''(0) = -14.3\ .$$

(a) Find the analytic solution, which will be used to validate the numerical solutions.
(b) Solve the initial value problem by first converting the original system into a system of first-order ODEs, then solving the latter using the following three methods ("Δx" is "h" in the textbook):

(I) Euler's explicit method, with $\Delta x = 0.1$
(II) Euler's explicit method, with $\Delta x = 0.03$
(III) Mid-point method, with $\Delta x = 0.1$

In all 3 cases, numerically integrate the system to $x = 1.5$ to find the solution over the interval of $0 \leq x \leq 1.5$. Plot the analytic solution and the numerical solutions using methods (I), (II), and (III) over the interval of $0 \leq x \leq 1.5$. Collect all four curves in one plot and clearly label the curves.

Note: For both Task 1 and 2, the numerical solutions are the main deliverables. An incorrect or missing analytic solution will be assessed a relatively minor deduction (~ 0.5 point).