MAE384, Spring 2018 Homework #6

Hard copy of report is due 12:15 PM on the due date. Computer codes used to complete the tasks should be included in the report.

Note: Uses of Matlab built-in functions for solving boundary value problems, such as **bvp4c** and **bvpinit**, are NOT allowed for this homework.

Task 0 (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of report**. If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the report has not helped anyone or received help from anyone on this assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the extent of collaboration. Please see related clarifications in Homework #1.

Task 1 (4 points)
Consider the following boundary value problem for $u(x)$ defined on the interval of $0 \le x \le 1$,

$$u'' + 64\, u = 0\ , \quad u(0) = -1\ , \ u(1) = 1$$ ("prime" is differentiation with respect to x, "d/dx")

(a) Find the analytic solution, which will be used to validate the numerical solutions.
(b) Solve the BVP using the finite-difference method. Specifically, use the 3-point central difference formula to approximate the second derivative: $u_i'' \approx (u_{i-1} - 2\, u_i + u_{i+1})/((\Delta x)^2)$. Obtain the numerical solution for the following two cases: (I) $\Delta x = 0.1$, (II) $\Delta x = 0.02$.

Plot the analytic solution and two numerical solutions over the interval of $0 \le x \le 1$. Collect all three curves in one plot and clearly label the curves.

Task 2 (5 points)
Consider the following boundary value problem for $u(x)$ defined on the interval of $0.5 \le x \le 3$,

$$x\, u'' + \left(1 - x^2\right) u' + \sin(x)\, u = 0\ , \quad u(0.5) = 3\ , \ u(3) = 1\ .$$

(Note that the left end point of the interval is $x = 0.5$, not $x = 0$.)

Solve the BVP using the finite-difference method. Specifically, use the 3-point central difference formula to approximate the second derivative: $u_i'' \approx (u_{i-1} - 2\, u_i + u_{i+1})/((\Delta x)^2)$, and use the 2-point forward difference formula to approximate the first derivative: $u_i' \approx (u_{i+1} - u_i)/(\Delta x)$. Obtain the numerical solution for the following two cases: (I) $\Delta x = 0.1$, (II) $\Delta x = 0.01$. Plot the two numerical solutions over the interval of $0.5 \le x \le 3$. Collect both curves in one plot and clearly label the curves. You do not need to find or show the analytic solution for this problem.