PLEASE READ THIS PAGE FIRST

A statement on collaboration is required for all reports, including those that are done independently without any collaboration. See instruction below on how to prepare the statement. Please read the rules before forming any collaboration for the homework. A violation of the rule(s) given in this page will be considered a violation of ASU's Academic Integrity Policy.

Rules on collaboration for homework:

(1) Collaboration is not allowed unless all involved follow rules (2)-(3) and unless the extent of collaboration is properly disclosed in a statement in the first page of the report for the assignment. See additional instruction below for the required content of the statement.

(2) For each assignment, each person can have maximum of one collaborator. Be aware that a collaborator's collaborator counts as a collaborator. For example, if Alice collaborates with Bob and Bob collaborates with Charles, Charles counts as a collaborator of Alice. All three violate the rule. In other words, collaboration can only be carried out within an isolated "team of two". Please talk to a potential collaborator to ensure that this rule is not violated before establishing any collaboration.

(3) In a legitimate collaboration, each individual must make a non-negligible contribution to the collaborative effort. Taking the solution and/or code from another student without making a reciprocal contribution to it is not allowed. To certify that a collaboration is legitimate, the submitter's contribution to the collaborative effort must be documented in the statement on collaboration.

The statement on collaboration

This statement is mandatory and must be placed in the beginning of the first page of report. If no collaboration occurred, simply state "**No collaboration**". This implies that the person submitting the report ***has not helped anyone or received help from anyone*** in the process of completing the assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the contribution by the submitter to the collaborative effort. Example:

Name of collaborator: Joe Smith	
Task(s), specific detail	Contribution to collaborative effort
Prob 1, matlab code	Developed codes with collaborator
Prob 3, mathematical derivation	Helped check mathematical equations

MAE 384, Spring 2020 Homework #1 (1 point ≈ 1% of the total score for the semester.)

Hard copy report is due at 12:15 PM on the due date. Please follow the rules on collaboration as described in the preceding page. **A statement on collaboration is required for all reports, including those that are produced independently. If there is no collaboration, write "No collaboration".** Please include the printout of computer code(s) in the report.

Each of the three problems requires that the given equation be solved using a specific method. No credit will be awarded if the equation is not solved by the designated method (with the code as proof). No credit will be awarded if the equation is solved by Matlab built-in function `fzero`, `solve`, or `vpasolve`.

Prob 1 (4 points)
Use **Bisection method** to find all solutions of the equation,

$$e^{[\sin(e^x)]} + 0.1x^2 = 2\,.$$

To receive full credit, the absolute error of each numerical solution must be smaller than 0.00001. Here, the absolute error is defined as $E = |X_N - X_S|$, where X_N and X_S are the numerical solution and true solution, respectively. The deliverable is a list of the solutions in ascending order, i.e., from the smallest to the largest.

Prob 2 (3 points)
(a) The following equation,

$$e^{-(x-1)^2} + 2\,e^{-(x-3)^2} - 0.1\,x^2 - 0.8 = 0\ ,$$

has four solutions that are all located within the interval of $0 \le x \le 3.5$. If **Newton's method** is used to seek the solutions, different choices of the initial guess could potentially lead to different solutions. Consider the 36 initial guesses, $x_0 = (0, 0.1, 0.2, 0.3, \ldots, 3.3, 3.4, 3.5)$. Use **Newton's method** to perform 100 iterations for each of those initial guesses. The deliverable is a list the solutions (as obtained at the end of 100 iterations) and their corresponding initial guesses in the following format:

$x_0 = 0$ solution = *.*******
$x_0 = 0.1$ solution = *.*******
$x_0 = 0.2$ solution = *.*******
...

(b) From the result of Part (a), one could see that with a given x_0 it is somewhat predictable which solution it will converge to. This breaks down when x_0 is close to a location where $f'(x) = 0$. (Here, "$f(x) = 0$" describes the original equation and $f'(x)$ is the derivative of $f(x)$.) To examine what is happening at those locations, repeat Part (a) but now consider the 11 initial guesses, $x_0 = (0.88, 0.89, 0.9, 0.91, \ldots, 1.06, 1.07, 1.08)$. These values fall within a short interval that encloses a point where $f'(x) = 0$, and the increment of x_0 is refined from 0.1 (as used in Part (a)) to 0.01. The deliverable of this part is the same as Part (a), i.e., a list of "initial guess vs. solution at 100 iterations". Briefly discuss the results from Part (a) and (b). In particular, address whether (and how) one can predict the particular solution that a given initial guess x_0 will converge to.

Since the codes used for Part (a) and (b) are almost identical, it suffices to submit the code for Part (a) in the report.

Prob 3 (3 points)
The equation,

$$5\,e^{-(x-2)^2} - x^4 + 1 = 0\,,$$

has one positive solution (around 1.48194) and one negative solution (around –1.00015). In this exercise, the original equation is rearranged into the form, $x = g(x)$, and **Fixed-point iterative method** is used to seek the solution.

(i) Given the initial guess of $x_0 = 1$, find a form of $g(x)$ such that the iterative process converges to the positive solution.

(ii) Given the initial guess of $x_0 = 1$, find a form of $g(x)$ such that the iterative process converges to the negative solution.

(iii) Given the initial guess of $x_0 = 10$, find a form of $g(x)$ such that the iterative process converges to the positive solution.

For case (i) and (ii), the key deliverable is the form of $g(x)$. No need to show detailed output of the iterative process.

For case (iii), in addition to giving the form of $g(x)$, please briefly explain how (based on what reasons) the "good choice" of $g(x)$ is obtained. Otherwise, no need to show detailed output of the iterative process.

Since the codes used for all cases are the same except in the detail of $g(x)$, for this problem please just provide the segment of the code that is common to all three cases, plus the portion of the code for the calculation of $g(x)$ (for the three $g(x)$'s in the three cases).