MAE384, Spring 2020 Homework #5

Please upload the report to Canvas as a single pdf or doc/docx file.
A statement on collaboration is required for all reports. This statement must be placed in the beginning of the first page of report. If no collaboration occurred, simply state "No collaboration". Please see related clarifications in the front page of Homework #1.

Note: Uses of Matlab built-in functions for solving ODEs, such as **ode23** and **ode45**, are NOT allowed for this homework.

Prob 1 (6 points)
For $u(x)$ defined on $x \geq 0$, consider the initial value problem,

$$\frac{du}{dx} = -x^2 u + e^{-\frac{x^3}{3}} \quad , \qquad u(0) = 1 \ .$$

(a) Find the analytic solution, which will be used to validate the numerical solutions. (You may use any methods or tools to obtain the analytic solution, but do describe in the report how you obtain it.)
(b) Solve the initial value problem using the following three methods ("Δx" is "h" in the textbook):

(I) Euler's implicit method, with $\Delta x = 0.1$
(II) Modified Euler method, with $\Delta x = 0.1$
(III) Third-order Runge-Kutta method, with $\Delta x = 0.1$

In each case, find the numerical solution over the interval of $0 \leq x \leq 2$. Plot the analytic solution and the numerical solutions using methods (I), (II), and (III) over the interval of $0 \leq x \leq 2$. Please collect all four curves in one plot and clearly label the curves. Please include computer code in the report. No code, no credit.

Prob 2 (4 points)
Consider the following initial value problem for $u(x)$ defined on $x \geq 0$,

$$u''' + 2u'' - u' - 2u = 0, \quad \text{("prime" is the derivative, d/d}x\text{)}$$

$$u(0) = 4 \ , \ u'(0) = -3 \ , \ u''(0) = 7 \ .$$

(a) Find the analytic solution, which will be used to validate the numerical solutions. (You may use any methods or tools to obtain the analytic solution, but do describe in the report how you obtain it.)
(b) Solve the initial value problem by first converting the original system into a system of first-order ODEs, then solving the latter using the following two methods ("Δx" is "h" in the textbook):

(I) Euler's explicit method, with $\Delta x = 0.1$
(II) Mid-point method, with $\Delta x = 0.1$

In each case, find the numerical solution over the interval of $0 \leq x \leq 1.5$. Plot the analytic solution and the numerical solutions using methods (I) and (II) over the interval of $0 \leq x \leq 1.5$. Collect all three curves in one plot and clearly label the curves. Please include computer code in the report. No code, no credit.

Note: For both Prob 1 and Prob 2, the numerical solutions are the main deliverables. An incorrect or missing analytic solution will be assessed a relatively minor deduction (~ 0.5 point).