MAE384, Spring 2020 Homework #6

Please upload the report to Canvas as a single pdf or doc/docx file. A statement on collaboration is required for all reports. This statement must be placed in the beginning of the first page of report. If no collaboration occurred, simply state "No collaboration". Please see related clarifications in the front page of Homework #1.

Note: Your report should include the computer code(s) written by yourself. Uses of Matlab built-in functions for solving boundary value problems, such as **bvp4c** and **bvpinit**, are NOT allowed for this homework. Otherwise, you may use Matlab functions such as backdivide ("\") or **inv** to solve a system of linear equations or invert a matrix.

Note: For any sinusoidal functions you might encounter in this homework, the argument of the function is always in radian.

Prob 1 (4 points)
Consider the following boundary value problem for $u(x)$ defined on the interval of $0 \le x \le 1$,

$$u'' + 25u = 0\ , \quad u(0) = 1 \quad u'(1) = 0.5\ . \quad \text{("prime" is differentiation with respect to } x\text{, "d/d}x\text{")}$$

Note that the boundary condition at $x = 1$ is imposed on the derivative of u.
(a) Find the analytic solution, which will be used to validate the numerical solutions. (You may use any methods/tools to obtain the analytic solution, but please describe in the report how the solution is obtained.)
(b) Solve the BVP using the finite-difference method. Specifically, use the 3-point second-order central difference formula to approximate the second derivative in the ODE: $u_i'' \approx (u_{i-1} - 2\,u_i + u_{i+1})/((\Delta x)^2)$. For the boundary condition at $x = 1$, use the 2-point first-order backward finite difference formula to approximate the first derivative: $u_i' \approx (u_i - u_{i-1})/(\Delta x)$. Obtain the numerical solution for the following two cases: (I) $\Delta x = 0.1$, (II) $\Delta x = 0.02$.

Plot the analytic solution and two numerical solutions over the interval of $0 \le x \le 1$. Collect all three curves in one plot and clearly label the curves.

Prob 2 (6 points)
Consider the following boundary value problem for $u(x)$ defined on the interval of $1 \le x \le 3$,

$$x\,u'' + (0.5 - x^2)\,u' + \sin(x)\,u = 0, \quad u(1) = 3\ ,\ u(3) = 2\ .$$

(Note that the left end point of the interval is $x = 1$, not $x = 0$.)

Solve the BVP using the finite-difference method. Specifically, use the 3-point second-order central difference formula to approximate the second derivative: $u_i'' \approx (u_{i-1} - 2\,u_i + u_{i+1})/((\Delta x)^2)$, and use the 2-point first-order forward difference formula to approximate the first derivative: $u_i' \approx (u_{i+1} - u_i)/(\Delta x)$. Obtain the numerical solutions for the following two cases: (I) $\Delta x = 0.1$, (II) $\Delta x = 0.01$. Plot the two numerical solutions over the interval of $1 \le x \le 3$. Collect both curves in one plot and clearly label the curves. You do not need to find or show the analytic solution for this problem.