## PLEASE READ THIS PAGE FIRST

Please include a Statement of Collaboration in the first page of your work. This statement is required even if you have done the work independently. Please read the rules before forming any collaboration for homework. A violation of the rule(s) given in this page will be considered a violation of ASU's Academic Integrity Policy.

## Rules on collaboration for homework:

(1) Collaboration is not allowed unless all involved follow rules (2)-(3) and unless the extent of collaboration is disclosed in a Statement of Collaboration. See an example below for the expected content of SoC.
(2) For each assignment, each person can have maximum of one collaborator. Be aware that a collaborator's collaborator counts as a collaborator. For example, if Alice collaborates with Bob and Bob collaborates with Charles, Charles counts as a collaborator of Alice. All three violate the rule. Please talk to a potential collaborator to ensure that this rule is not violated before establishing any collaboration.
(3) In a legitimate collaboration, each individual must make a non-negligible contribution to the collaborative effort. Taking the solution or code from another student without making a reciprocal contribution to it is not allowed. To certify that a collaboration is legitimate, the submitter's contribution to the collaborative effort must be documented in the Statement of Collaboration.

## The Statement of Collaboration

This statement is mandatory and must be placed in the beginning of the first page of your submitted work. If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the work has not helped anyone or received help from anyone in the process of completing the assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the contribution by the submitter to the collaborative effort. Example:

| Name of collaborator: Joe Smith |  |
| :--- | :--- |
| Task(s), specific detail | Contribution to collaborative effort |
| Prob 1, matlab code | Developed codes with collaborator |
| Prob 3, mathematical derívation | Helped check mathematical equations |

MAE 384 Spring 2022 Homework \#1 ( 1 point $\approx 1 \%$ of the total score for the semester.)
A statement of collaboration is required. If there is no collaboration, write "No collaboration". Each of the three problems requires the use of a specific method. No credit will be given if the equation is not solved by the designated method (with the accompanying computer code as proof). No credit will be given if the equation is solved by Matlab built-in function fzero, solve, or vpasolve. Please include computer codes in your work.

Problem 1 (4 points)
Use Bisection method to find all solutions of the equation,

$$
\cosh (x)+2\left(x^{3}-x\right) \sin (5 x)-0.7=0
$$

To receive full credit, the absolute error of each numerical solution must be smaller than 0.00001 . The absolute error is defined as $E=\left|X_{N}-X_{S}\right|$, where $X_{N}$ and $X_{S}$ are the numerical solution and true solution, respectively. The deliverables are the code and a list of the solutions in ascending order, i.e., from the smallest to the largest.

## Problem 2 (3 points)

Consider the equation,

$$
f(x)=0
$$

where $f(x)$ is given as

$$
f(x) \equiv x e^{0.1 x}+x^{3}-6 x^{2}+7 x-2
$$

The equation has three solutions that are all located within the interval of $0 \leq x \leq 5$. If Newton's method is used to seek the solutions, different choices of initial guess may lead to different solutions. Consider the 51 initial guesses, $x_{0}=(0,0.1,0.2,0.3, \ldots, 4.8,4.9,5)$. Use Newton's method to perform 100 iterations for each initial guess. The deliverables are the code and its output as a list of initial guess vs. the solution it converges to, in the following format:

$$
\begin{array}{ll}
x_{0}=0 & \text { solution }=* . * * * * * * * \\
x_{0}=0.1 & \text { solution }=* . * * * * * * * \\
x_{0}=0.2 & \text { solution }=* . * * * * * * *
\end{array}
$$

The list essentially defines a function, $S(x)$, where the input $x$ is the initial guess and $S$ is the solution corresponding to the initial guess. As an additional deliverable, make a plot of $S(x)$ and superimpose $f(x)$ and $f^{\prime}(x)$ on it $\left(f^{\prime}(x)\right.$ is the derivative of $\left.f(x)\right)$. To visualize the locations of zeros of $f(x)$, the plot should also include the zero line whose intersections with $f(x)$ are the solution of $f(x)=0$. Please collect all 4 curves $\left(S(x), f(x), f^{\prime}(x)\right.$, and zero line) in one plot and label them clearly. The plot should cover the range of $0 \leq x \leq 5$. Based on the plot, briefly discuss the relation between initial guess and final converged solution.

Problem 3 (3 points)
The equation,

$$
e^{\left(x^{2}-2 x+1\right)}+1=e^{\sin (x)}
$$

has two solutions. The first solution, $X_{1}$, is around 0.7955 . The second solution, $X_{2}$, is around 1.7231. In this exercise, we rewrite the original equation into the form, $x=g(x)$, and solve it by Fixed-point iterative method. Complete the two tasks:
(i) Given the initial guess of $x_{0}=1$, find a $g(x)$ such that the iterative process converges to $X_{1}$.
(ii) Given the initial guess of $x_{0}=2.5$, find a $g(x)$ such that the iterative process converges to $X_{2}$.

For each case, the deliverables are the form of $g(x)$, and computer code and output that demonstrates that the iterative process convergences to the desired solution.

For this problem, full credit will be given if the iterative process stays on the real line. In other words, the outcome of successive iterations $x_{0}, x_{1}, x_{2}, \ldots, x_{\mathrm{N}}$ are all real. Expect a minor deduction if the process strays from the real line (i.e., some of the $x_{\mathrm{n}}$ are complex), even if it eventually converges to the desired solution. (We impose this requirement to make the exercise slightly more challenging.) To confirm that all of $\left\{x_{\mathrm{n}}\right\}$ are real, please output both the real part and imaginary part of $x_{n}$.

