MAE384 Spring 2022 Homework 1 Solution

The strategy for coding for Problem 1 and 2 will be discussed in class.

Problem 1

The reference solution produced by instructor satisfies |true error| < 0.00001. As such, student's solution will be considered satisfactory if it is within ±0.00002 of instructor's solution.

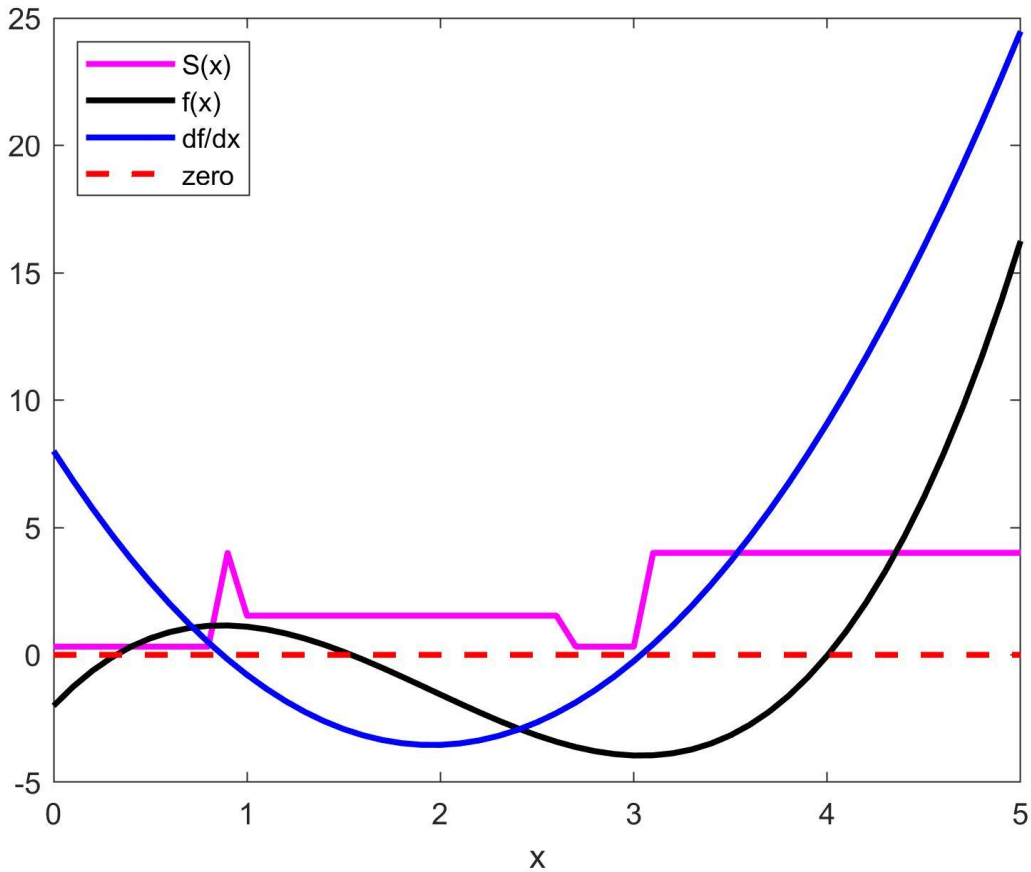
```
solution  1 = -7.2830898
solution  2 = -7.1245664
solution  3 = -6.1733398
solution  4 = -5.7421289
solution  5 = -4.9651602
solution  6 = -4.4490977
solution  7 = -3.7277773
solution  8 = -3.1809961
solution  9 = -2.4712852
solution 10 = -1.9388320
solution 11 = -0.5064492
solution 12 = -0.1986836
solution 13 =  0.1986836
solution 14 =  0.5064492
solution 15 =  1.9388320
solution 16 =  2.4712852
solution 17 =  3.1809961
solution 18 =  3.7277773
solution 19 =  4.4490977
solution 20 =  4.9651602
solution 21 =  5.7421289
solution 22 =  6.1733398
solution 23 =  7.1245664
solution 24 =  7.2830898
```

Problem 2

```
x0 = 0.0  solution =  0.3224834372
x0 = 0.1  solution =  0.3224834372
x0 = 0.2  solution =  0.3224834372
x0 = 0.3  solution =  0.3224834372
x0 = 0.4  solution =  0.3224834372
x0 = 0.5  solution =  0.3224834372
x0 = 0.6  solution =  0.3224834372
```

```
x0 = 0.7  solution =  0.3224834372
x0 = 0.8  solution =  0.3224834372
x0 = 0.9  solution =  4.0035893010
x0 = 1.0  solution =  1.5396885941
x0 = 1.1  solution =  1.5396885941
x0 = 1.2  solution =  1.5396885941
x0 = 1.3  solution =  1.5396885941
x0 = 1.4  solution =  1.5396885941
x0 = 1.5  solution =  1.5396885941
x0 = 1.6  solution =  1.5396885941
x0 = 1.7  solution =  1.5396885941
x0 = 1.8  solution =  1.5396885941
x0 = 1.9  solution =  1.5396885941
x0 = 2.0  solution =  1.5396885941
x0 = 2.1  solution =  1.5396885941
x0 = 2.2  solution =  1.5396885941
x0 = 2.3  solution =  1.5396885941
x0 = 2.4  solution =  1.5396885941
x0 = 2.5  solution =  1.5396885941
x0 = 2.6  solution =  1.5396885941
x0 = 2.7  solution =  0.3224834372
x0 = 2.8  solution =  0.3224834372
x0 = 2.9  solution =  0.3224834372
x0 = 3.0  solution =  0.3224834372
x0 = 3.1  solution =  4.0035893010
x0 = 3.2  solution =  4.0035893010
x0 = 3.3  solution =  4.0035893010
x0 = 3.4  solution =  4.0035893010
x0 = 3.5  solution =  4.0035893010
x0 = 3.6  solution =  4.0035893010
x0 = 3.7  solution =  4.0035893010
x0 = 3.8  solution =  4.0035893010
x0 = 3.9  solution =  4.0035893010
x0 = 4.0  solution =  4.0035893010
x0 = 4.1  solution =  4.0035893010
x0 = 4.2  solution =  4.0035893010
x0 = 4.3  solution =  4.0035893010
x0 = 4.4  solution =  4.0035893010
x0 = 4.5  solution =  4.0035893010
x0 = 4.6  solution =  4.0035893010
x0 = 4.7  solution =  4.0035893010
x0 = 4.8  solution =  4.0035893010
x0 = 4.9  solution =  4.0035893010
x0 = 5.0  solution =  4.0035893010
```

Plot:



Problem 3

There are many choices of g(x) that will work for each of (a) and (b). Examples:

(a) For x0 = 1, choose $g(x) \equiv arcsin(ln(exp(x^2 - 2x + 1) + 1))$

```
xN = 1; N = 20;
for n = 1:N
    xNp1 = asin(log(exp((xN-1)^2)+1));
    xN = xNp1;
    fprintf('%3u --> sol = %13.10e %13.10e \n',n,real(xN),imag(xN))
end
```

```
  1 --> sol = 7.6584619482e-01 0.0000000000e+00
  2 --> sol = 8.0515331590e-01 0.0000000000e+00
  3 --> sol = 7.9278390359e-01 0.0000000000e+00
  4 --> sol = 7.9640511799e-01 0.0000000000e+00
  5 --> sol = 7.9531959369e-01 0.0000000000e+00
  6 --> sol = 7.9564278219e-01 0.0000000000e+00
  7 --> sol = 7.9554636250e-01 0.0000000000e+00
  8 --> sol = 7.9557511065e-01 0.0000000000e+00
  9 --> sol = 7.9556653764e-01 0.0000000000e+00
 10 --> sol = 7.9556909406e-01 0.0000000000e+00
 11 --> sol = 7.9556833174e-01 0.0000000000e+00
 12 --> sol = 7.9556855906e-01 0.0000000000e+00
 13 --> sol = 7.9556849128e-01 0.0000000000e+00
 14 --> sol = 7.9556851149e-01 0.0000000000e+00
 15 --> sol = 7.9556850546e-01 0.0000000000e+00
 16 --> sol = 7.9556850726e-01 0.0000000000e+00
 17 --> sol = 7.9556850672e-01 0.0000000000e+00
 18 --> sol = 7.9556850688e-01 0.0000000000e+00
 19 --> sol = 7.9556850684e-01 0.0000000000e+00
 20 --> sol = 7.9556850685e-01 0.0000000000e+00
```

(b) For x0 = 2.5, choose $g(x) \equiv x - 0.05\, f(x)$ (where $f(x) = 0$ is the original equation to solve).

```
xN = 1; N = 60;
for n = 1:N
    xNp1 = xN-0.05*(exp((xN-1)^2)+1-exp(sin(xN)));
    xN = xNp1;
    fprintf('%3u --> sol = %13.10e %13.10e \n',n,real(xN),imag(xN))
end
```

```
  1 --> sol = 2.0665800577e+00 0.0000000000e+00
  2 --> sol = 1.9811138026e+00 0.0000000000e+00
  3 --> sol = 1.9252790409e+00 0.0000000000e+00
  4 --> sol = 1.8852971882e+00 0.0000000000e+00
  5 --> sol = 1.8552221386e+00 0.0000000000e+00
  6 --> sol = 1.8318839443e+00 0.0000000000e+00
  7 --> sol = 1.8133806237e+00 0.0000000000e+00
  8 --> sol = 1.7984795396e+00 0.0000000000e+00
  9 --> sol = 1.7863369695e+00 0.0000000000e+00
 10 --> sol = 1.7763513143e+00 0.0000000000e+00
 … skip some output …
 50 --> sol = 1.7232485283e+00 0.0000000000e+00
 51 --> sol = 1.7232338218e+00 0.0000000000e+00
 52 --> sol = 1.7232212100e+00 0.0000000000e+00
 53 --> sol = 1.7232103943e+00 0.0000000000e+00
 54 --> sol = 1.7232011190e+00 0.0000000000e+00
 55 --> sol = 1.7231931645e+00 0.0000000000e+00
 56 --> sol = 1.7231863429e+00 0.0000000000e+00
 57 --> sol = 1.7231804926e+00 0.0000000000e+00
 58 --> sol = 1.7231754754e+00 0.0000000000e+00
 59 --> sol = 1.7231711727e+00 0.0000000000e+00
 60 --> sol = 1.7231674827e+00 0.0000000000e+00
 61 --> sol = 1.7231643181e+00 0.0000000000e+00
```