MAE384, Spring 2022 Homework #7

A statement of collaboration is required. For this assignment, uses of Matlab built-in functions for solving boundary value problems, such as **bvp4c** and **bvpinit**, are NOT allowed. Otherwise, you may use Matlab functions such as backslash ("\") or **inv** to solve a system of linear equations or invert a matrix.

Note: For any sinusoidal functions you might encounter in this homework, the argument of the function is always in radian.

Problem 1 (5 points)
Consider the following boundary value problem for $u(x)$ defined on the interval of $0 \le x \le 1$,

$$u'' + 9u = 0 \ , \quad u(0) = 0.5 \quad u'(1) = 0 \ . \quad \text{("prime" is differentiation with respect to } x\text{, "d/d}x\text{")}$$

Note that the boundary condition at $x = 1$ is imposed on the derivative of u.
(a) Find the analytic solution, which will be used to validate the numerical solutions. (You may use any methods/tools to obtain the analytic solution, but please describe in the report how the solution is obtained.)
(b) Solve the BVP using the finite-difference method. Specifically, use the 3-point second-order central difference formula to approximate the second derivative in the ODE: $u_i'' \approx (u_{i-1} - 2\,u_i + u_{i+1})/((\Delta x)^2)$. For the boundary condition at $x = 1$, use the 2-point first-order backward finite difference formula to approximate the first derivative: $u_i' \approx (u_i - u_{i-1})/(\Delta x)$. Obtain the numerical solution for the two cases: (I) $\Delta x = 0.2$, (II) $\Delta x = 0.05$.

Plot the analytic solution and two numerical solutions over the interval of $0 \le x \le 1$. Collect all three curves in one plot and clearly label the curves.

Problem 2 (5 points)
Consider the following boundary value problem for $u(x)$ defined on the interval of $1 \le x \le 3$,

$$x\,u'' + (1 - x^2)\,u' + \cos(x)\,u = 0, \quad u(1) = 4\,,\ u(3) = 2\,.$$

(Note that the left end point of the interval is $x = 1$, not $x = 0$.)

Solve the BVP using the finite-difference method. Specifically, use the 3-point second-order central difference formula to approximate the second derivative: $u_i'' \approx (u_{i-1} - 2\,u_i + u_{i+1})/((\Delta x)^2)$, and use the 2-point first-order forward difference formula to approximate the first derivative: $u_i' \approx (u_{i+1} - u_i)/(\Delta x)$. Obtain the numerical solutions for the two cases: (I) $\Delta x = 0.2$, (II) $\Delta x = 0.02$. Plot the two numerical solutions over the interval of $1 \le x \le 3$. Collect both curves in one plot and clearly label the curves. You do not need to find or show the analytic solution for this problem.