

MAE384 Homework #3

1. Solve the equation,

$$\begin{pmatrix} 3 & 8 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix},$$

using the LU decomposition method. No credit if you use any other methods, even with the correct final solution. As always, please show your work. **3 points**

2. Solve the equation,

$$\begin{pmatrix} 0 & 1 & 3 \\ 2 & 3 & -1 \\ -1 & -1 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \\ 5 \end{pmatrix},$$

using the Gauss elimination method. Apply pivoting if necessary. **2 points**

3. (a) Find the condition number of the matrix,

$$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 2 & 7 \end{pmatrix}.$$

Use the *Euclidean norm* (Eq. (4.76) in G&S textbook) where the norm of a matrix needs to be evaluated. (b) Based on the result of (a), would you consider the equation, $\mathbf{Ax} = \mathbf{b}$ (\mathbf{b} is an arbitrary vector), ill-conditioned? **2 points**

4. Solve the system of linear equations,

$$\begin{aligned} 6x_1 + x_2 + x_3 &= 4 \\ x_1 + 6x_2 + x_3 &= 4 \\ x_1 + x_2 + 6x_3 &= 4, \end{aligned}$$

using the Gauss-Seidel iteration method with initial guess of $(x_1, x_2, x_3) = (0,0,0)$. Perform 3 iterations, i.e., update each of x_1 , x_2 , and x_3 3 times. **3 points**