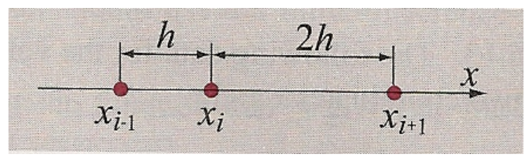
MAE 384 Homework 5

1. (a) Given the function

$$f(x) = e^{-x}\sin(x)\ ,$$

evaluate the first derivative, $f'(x)$, at $x = 1$ using the *three-point backward scheme* (2nd formula from the top of Table 6-1 in p. 248 of G&S textbook). (b) Same as (a) but using the *four-point central difference scheme* (4th formula from the top of Table 6-1). (c) Evaluate the second derivative, $f''(x)$, at $x = 1$ using the *four-point backward scheme* (4th formula in the middle panel of Table 6-1). Use $h = 0.5$ for all calculations in (a)-(c). (d) compare your results in (a), (b), and (c) with the exact values obtained from the analytic expressions of $f'(x)$ and $f''(x)$. Evaluate the true relative errors for the three cases. **2 points**

2. (Modified from Prob 6.6 in G&S textbook) Derive a finite difference formula for the second derivative, $f''(x_i)$, that depends on the values of $f(x)$ at three points x_{i-1}, x_i, and x_{i+1}, where the spacing is such that $x_i - x_{i-1} = h$ and $x_{i+1} - x_i = 2h$. See the illustration below. What is the order of the truncation error of your formula ($O(h)$, $O(h^2)$, etc.)? **2 points**



3. All of the formula in Table 6-1 have a truncation error of $O(h)$, $O(h^2)$, or $O(h^4)$. Try to derive a finite difference formula for the first derivative, $f'(x_i)$, that has a truncation error of $O(h^3)$. Must show your procedure to receive credit. **3 points**
[Hint: Try to combine the Taylor series expansion at three neighboring points of x_i, for example (x_{i-1}, x_{i+1}, x_{i+2}) or (x_{i-2}, x_{i-1}, x_{i+1}). Different choices of the neighboring points will lead to different formula. You only need to work on one of them.]

4. Evaluate the integral

$$I = \int_0^6 \sin(4x)\,dx\ ,$$

using (i) Trapezoidal method, (ii) Simpson's 1/3 method, both with $h = 0.5$. Compare the numerical results with the exact value obtained from the analytic expression of I. Which numerical method produces the better answer? **2 points**