MAE 384 HW2

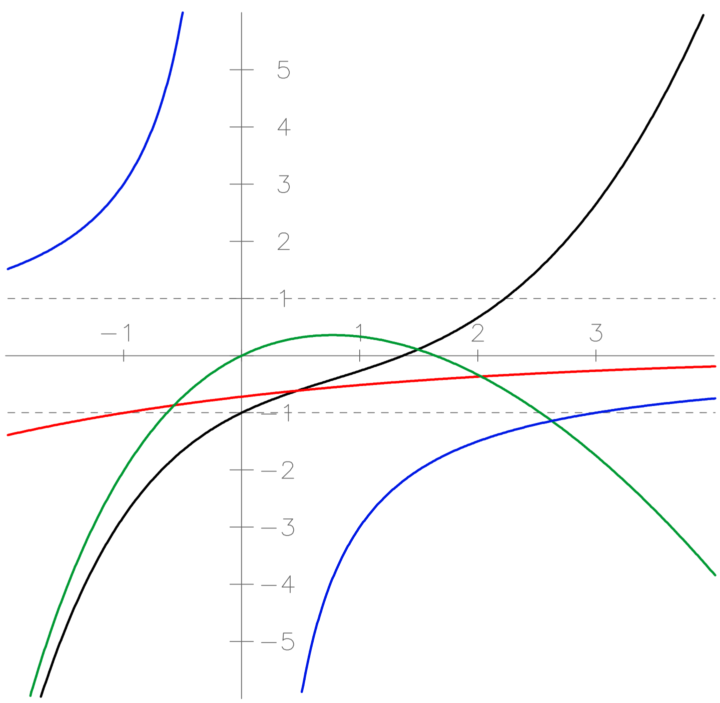
Comments

Prob 1 is straightforward. Note that there are <u>two</u> solutions. See attached sample solution.

Prob 2

There are many ways to rewrite the original equation into the form, $x = \mathrm{g}(x)$, for the iterative procedure. Just to name three: **(A)** $\mathrm{g}(x) = (10\exp(-x))^{1/3}$ **(B)** $\mathrm{g}(x) = x - 0.1\,x^3 + \exp(-x)$ **(C)** $\mathrm{g}(x) = -\ln(0.1\,x^3)$

As indicated by Eq. (3.30) in the textbook, the process is expected to converge only if $|\mathrm{g}'(x)| < 1$ in the neighborhood of the solution. Some of you first noted, by inspecting the plot of $f(x) = 0.1\,x^3 - \exp(-x)$, that the solution is located between 1 and 2. You then showed that $|\mathrm{g}'(1)| < 1$ and $|\mathrm{g}'(2)| < 1$ for your choice of $\mathrm{g}(x)$ before performing the iteration. That was a sensible practice, but one can make it even more convincing by plotting $\mathrm{g}'(x)$ and $f(x)$ together to show that $|\mathrm{g}'(x)| < 1$ right where the solution is located. In the following figure, the black curve is $f(x)$, and red, green, and blue curves are $\mathrm{g}'(x)$ for (A), (B), and (C). The two dashed lines indicate the bounds of 1 and -1. Clearly, (A) and (B) are viable choices while (C) is not.



Almost everyone who got it right chose form (A) - see attached sample solution. Only two persons chose to use form (B). Note that by using (B), we have $f(x) = x - \mathrm{g}(x) \Rightarrow f(x_i) = x_i - \mathrm{g}(x_i) = x_i - x_{i+1}$. (The last step used the relation, $x_{i+1} = \mathrm{g}(x_i)$, from the fixed point iterative procedure.) Thus, in this case $|f(x_i)|$ is immediately known as a by-product of the iterative process.

Sample solution, Prob 1 (Thanks to Michael Van Osten)

1) Solve the equation $0.5x^2 - \sin(x) + 0.1 = 0$

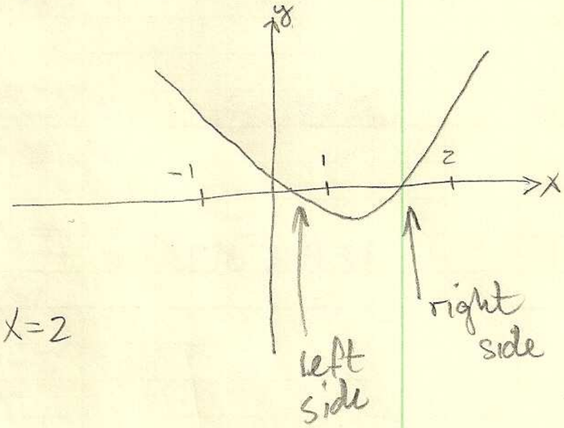
Find all solutions using Newtons Methods.

Solve to accuracy $|f(x_n)| \leq .0001$

Using my TI-84 Plus calculator, I graphed the function $.5x^2 - \sin(x) + .1 = 0$.

By the graph, the solution for zeros should be around $x=0$ & $x=1$.

Knowing this, my initial guess for each solution will be $x=-1$ and $x=2$



$x_1 = -1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = (-1) - \frac{(.5(-1)^2) - \sin(-1) + .1}{(-1) - \cos(-1)} = -.064163587$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = (-.064163587) - \frac{.5x_2^2 - \sin x_2 + .1}{(x_2) - \cos(x_2)} = .0922973333$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = (.0922973333) - \frac{.5x_3^2 - \sin x_3 + .1}{(x_3) - \cos(x_3)} = .1056828036$$

So, $f(.1056828036) = 9.824005309 \times 10^{-5}$

this solution is well within our accuracy tolerance of $|f(x_n)| < .0001$. By choosing $x_1 = -1$ to start, this allowed the method to converge

$\boxed{x_n = .1056828036 \text{ left side}}$

continued to next page

(Prob 1 continued)

1 cont'd

$x_1 = 2$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = (2) - \frac{.5(x_1^2) - \sin(x_1) + .1}{x_1 - \cos(x_1)} = 1.507189482$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = (1.507189482) - \frac{.5x_2^2 - \sin x_2 + .1}{x_2 - \cos x_2} = 1.342442948$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = (1.342442948) - \frac{.5x_3^2 - \sin x_3 + .1}{x_3 - \cos x_3} = 1.318821858$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = (1.318821858) - \frac{.5x_4^2 - \sin x_4 + .1}{x_4 - \cos x_4} = 1.317676931$$

$$f(1.317676931) = 2.8872321 \times 10^{-7}$$

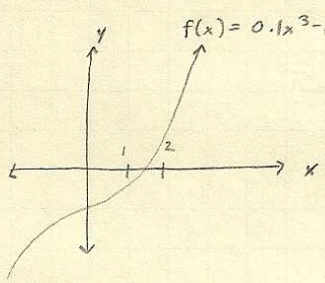
So, for the 2nd zero (right side)

$$\boxed{x_n = 1.317676931 \text{ right side}}$$

Sample solution, Prob 2 (Thanks to Katrin Passlack)

2. Solve $0.1x^3 - e^{-x} = 0$ using the fixed point iteration method

$|f(x_N)| < 0.01$



$x = 1$ $f(x) = -0.267879$ → sign change

$x = 2$ $f(x) = 0.664665$

Case #1

$0 = 0.1x^3 - e^{-x}$

$0.1x^3 = e^{-x}$

$\ln(0.1x^3) = \ln e^{-x}$

$\ln(0.1x^3) = -x$

$x = g(x) = -\ln(0.1x^3)$

$g'(x) = -\frac{3}{x}$

$g'(1) = -3$

$g'(2) = -3/2 = -1.5$

→ does not converge $|g'(x)| > 1$

Case #2

$0.1x^3 = e^{-x}$

$x^3 = \frac{e^{-x}}{0.1}$

$x = \left(\frac{e^{-x}}{0.1}\right)^{1/3}$

$x = g(x) = \left(\frac{e^{-x}}{0.1}\right)^{1/3} = (10e^{-x})^{1/3}$

$g'(x) = \frac{1}{3}(-10e^{-x})^{1/3}$

$g'(1) = -0.514576$

$g'(2) = -0.368708$

→ converges $|g'(x)| < 1$

$x_1 = 1$

$x_2 = g(x_1) = (10e^{-1})^{1/3} = 1.54372$

$x_3 = g(x_2) = (10e^{-1.54372})^{1/3} = 1.28783$

$x_4 = g(x_3) = (10e^{-1.28783})^{1/3} = 1.4025$

$x_5 = g(x_4) = (10e^{-1.4025})^{1/3} = 1.3499$

$x_6 = g(x_5) = (10e^{-1.3499})^{1/3} = 1.37377$

$f(x_1) = -0.267879$

$f(x_2) = 0.154294$

$f(x_3) = -0.062285$

$f(x_4) = 0.029888$

$f(x_5) = -0.013284$

$f(x_6) = 0.006117 < 0.01$ (tolerance)

$x_N = 1.3737744268464$

$\boxed{\text{Solution: } x_N = 1.37377}$