MAE502 Spring 2010 Homework #5

Problem 1 (5 points)

(a) Solve the heat equation with an internal source,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + Q(x,t) \quad , \quad Q(x,t) \equiv -\mathrm{B}\cos(2.5\,\pi\, x)\cos(20\, t) \quad \text{(B is a constant)},$$

for $u(x,t)$ defined within the domain of $0 \le x \le 1$ and $t \ge 0$, given the following boundary conditions

(i) $u_x(0, t) = 0$ (Note: $u_x \equiv \partial u/\partial x$)
(ii) $u(1, t) = 0$
(iii) $u(x,0) = 3\cos(0.5\,\pi\, x) + 2\cos(2.5\,\pi\, x)$,

(b) For each of the three cases with B = 0, 50, and 500, evaluate and plot your solutions for t = 0, 0.02, 0.1, and 0.5. <u>Discuss your results</u>.

Note that the b.c.'s (i) and (ii) are identical to those for HW2 Prob 1. The eigenvalues and eigenfunctions in x from that problem can be recycled here; no need to repeat the detail.

Prob 2 (2 points)

Consider the eigenvalue problem,

$$\sin(x)\, u'' + \cos(x)\, u' + e^x\, u = \lambda\, x^2\, u \ ,$$

with the boundary conditions,

(i) $u'(0.5) = 0$, (ii) $u(1.5) = 0$,

where $u'' \equiv d^2u/dx^2$, $u' \equiv du/dx$, and λ is the eigenvalue. Are the eigenfunctions of this problem orthogonal (in the sense defined in Sec. 5.3 in the textbook)? That is, are any two eigenfunctions $u_p(x)$ and $u_q(x)$ corresponding to eigenvalues p and q orthogonal to each other if p ≠ q? If no, explain why. If yes, describe what the orthogonality relation for $u_p(x)$ and $u_q(x)$ should be for this system. You do not have to explicitly solve for the eigenvalues and eigenfunctions to reach the conclusion.